Muon Capture in Deuterium

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Abstract

Model dependence of the capture rates of the negative muon capture in deuterium is studied starting from potential models and the weak two-body meson exchange currents constructed in the tree approximation and also from an effective field theory. The tree one-boson exchange currents are derived from the hard pion chiral Lagrangians of the $N\Delta\pi\rho\omega a_1$ system. If constructed in conjunction with the one-boson exchange potentials, the capture rates can be calculated consistently. On the other hand, the effective field theory currents, constructed within the heavy baryon chiral perturbation theory, contain a low energy constant $\hat{d}^R$ that cannot be extracted from data at the one-particle level nor determined from the first principles. Comparative analysis of the results for the doublet transition rate allows us to extract the constant $\hat{d}^R$.

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I. INTRODUCTION

The study of the weak interaction in the deuterium at low energies is an important topic. Since the structure of the nucleon-nucleon (NN) force is at present well understood and the weak vector one- and two-nucleon interactions and the weak axial one-nucleon interaction are also well known, it can provide the most reliable information on the structure of the space component of the weak axial meson exchange currents (MECs). This component contributes notably into the capture rates for the reaction of the negative muon capture in deuterium

$$\mu^- + d \longrightarrow \nu_\mu + n + n \,, \tag{1.1}$$

from the hyperfine μd states.
In reaction (1.1), the stopped muons are captured from the doublet or quadruplet hyperfine states with total angular momentum F=1/2 or 3/2, respectively. The corresponding capture rates, $\Lambda_{1/2}$ and $\Lambda_{3/2}$, were calculated in the past by many authors (for the references, see [1, 2]). Since $\Lambda_{3/2} \approx 10\ \mathrm{s}^{-1}$ and $\Lambda_{1/2} \approx 400\ \mathrm{s}^{-1}$, only the doublet capture rate is of practical interest.
Until recently, the capture rates were calculated with the deuteron and neutron-neutron wave functions derived from the variety of nuclear potentials of the first generation and with the MECs constructed in the tree approximation. The systematic investigation of the structure of the operator of the tree weak axial MECs based on the chiral Lagrangians was started in Ref. [3]. These MECs were applied in calculations of the capture rates and neutron-neutron spectra for reaction (1.1)in Refs. [4–7]. The obtained values of the doublet capture rate $\Lambda_{1/2}$ were in the interval $398\ \mathrm{s}^{-1}$ - $416\ \mathrm{s}^{-1}$, which was considered in a reasonable agreement with the measured value $\Lambda_{1/2} = 409 \pm 40\ \mathrm{s}^{-1}$ [8], in contrast to another measurement [9] providing somehow larger $\Lambda_{1/2} = 470 \pm 29\ \mathrm{s}^{-1}$. Let us note that in these calculations the MECs are of the one-boson exchange form. If used in conjunction with the nuclear wave functions generated from one-boson exchange potentials (OBEPs), all the parameters of the calculations are fixed. The same space component of the weak axial MECs induces other very important weak processes in the two-nucleon system, such as the weak neutrino- and antineutrino-deuteron disintegration and the proton-proton fusion. In the terrestrial conditions only the reaction (1.1) has the real hope to be studied experimentally with an accuracy of ≈ 1 % in the near future [10]. This is challenge also for theorists to improve the calculations of the capture rates. Let us note that the above described approach was highly successful in describing the electroweak processes in nuclei at low and intermediate energies [11, 12]. We shall call it the Tree Approximation Approach (TAA). It is also called the Standard Nuclear Physics Approach [13].
A more fundamental approach to the study of electroweak processes in nuclei was possible after the appearance of effective field theory (EFT) [14, 15]. In this way one constructs also terms of higher order, including loops besides the leading order of a current (potential) operator. The expansion is done in a perturbative parameter $q/\Lambda_\chi << 1$, where q is a quantity characterizing the system (momentum, energy,...), which is small in comparison with the heavy scale Λ_χ.
Recently, the reaction (1.1) was considered in two varieties of such an EFT. In Ref.[16], $\Lambda_{1/2}$ was calculated within the framework of a pionless EFT. In this approach, an unknown low energy constant (LEC) $L_{1,A}$ enters the weak axial MECs. It cannot be extracted from the one-body processes nor derived from fundamental considerations. In our opinion, the application of the pionless EFT to the reaction (1.1) is doubtful, because in this process

the contribution from the pion pole (the induced pseudoscalar) is important. Since the pion is absent, Chen *et al.* [16] introduced the pion pole by hands which is a misconception. Moreover, the heavy scale turns out to be of the order of the pion mass m_π, $\Lambda_\chi \sim m_\pi \sim$ 140 MeV, whereas the value of the momentum transfer q is of the order of the muon mass m_μ, $q \sim m_\mu \sim 100$ MeV. The calculated prediction for $\Lambda_{1/2}$ for reasonable values of $L_{1,A} \sim 5-6$ (see FIG. 3 and TABLE I of Ref.[16]) is $\Lambda_{1/2} \sim 370-380\,\mathrm{s}^{-1}$ MeV, which is by $\sim$ 5 - 10 % less than the results obtained within the above discussed TAA.

In Ref.[13], the muon capture in deuterium was studied within a 'hybrid' approach: the current operator was constructed within the framework of a heavy baryon EFT and the nuclear wave functions were obtained from the Schroedinger equation solved with the second generation Argonne v_{18} potential [17]. The weak axial MECs operator of the heavy baryon EFT contains the one-pion exchange part and also a (contact) short-range term, the strength of which is given by the LEC $\hat{d}^R$, that should be fixed by data. In Ref.[13], $\hat{d}^R$ is taken from the analysis of the triton β-decay rate [18] to predict the $\Lambda_{1/2}(^1S_0)$ for the transition $d \to {}^1S_0$. Let us note that the value of $\hat{d}^R$ is correlated with the cutoff Λ entering a Gaussian regulator (strong form factor). Since the Argonne potential is not a OBEP containing such a form factor, the value of Λ is not fixed by the NN scattering data. Rather, the extraction of $\hat{d}^R$ is made for three values $\Lambda = 500$ MeV, 600 MeV and 800 MeV. For these values of $\hat{d}^R$ and Λ, the doublet capture rate calculated in Ref. [13] for the channel $d \to {}^1S_0$, $\Lambda_{1/2}(^1S_0) = 245$ s^{-1}, is by $\sim$ 5.5 % less than the value $\Lambda_{1/2}(^1S_0) = 259$ s^{-1}, obtained in the TAA calculations [7].

Here we calculate the capture rates for the reaction (1.1) within both the TAA and the hybrid calculations. In both methods the basic symmetries of quantum chromodynamics, such as the gauge symmetry and the spontaneously broken chiral symmetry, are reflected. The nuclear wave functions are obtained by solving the Schroedinger equation

$$H\Psi = E\Psi, \quad H = T + V\,, \tag{1.2}$$

where T is the the kinetic energy operator and V is the NN potential. Since our TAA weak currents, constructed from chiral invariant Lagrangians in Refs. [3, 4, 6, 19, 20], are of the OBE type, we use OBEPs in Eq. (1.2), too. Such potentials are successfully applied in the TAA for describing nuclear phenomena at low energies. This modeling of the nuclear force is a surrogate of calculations based directly on the quark-gluon dynamics because of the non-perturbative feature of the quantum chromodynamics in this region of energies. The well known first generation Bonn OBEPs [21, 22] contain the exchanges of scalar, pseudoscalar and vector bosons. In the BNN vertices, phenomenological form factors of the form

$$F_\alpha(\vec{q}^{\,2}) = \left(\frac{\Lambda_\alpha^2 - m_\alpha}{\Lambda_\alpha^2 + \vec{q}^{\,2}}\right)^{n_\alpha} \tag{1.3}$$

are applied, with $\vec{q}$ the three-momentum transfer, Λ_α the so called cutoff mass and n_α=1 or 2 depending on the specific couplings. These form factors reflect the extended structure of the nucleon. Later on, high quality second generation CD-Bonn OBEP appeared [23], containing the BNN form factors of the type (1.3), too. Another type of OBEPs has been constructed by the Nijmegen group [24]. These potentials contain the Gaussian strong form factors,

$$F_{BNN}(\vec{q}^{2}) = e^{-\vec{q}^{2}/2\Lambda_B^2}\,, \tag{1.4}$$

where the cutoffs Λ_B are extracted from the fit to the NN scattering data.

In order to make our calculations of the capture rates consistent, we employ in Eq. (1.2) the Nijmegen OBEPs and use the Gaussian form factors (1.4) and couplings entering these potentials both in our weak and EFT MECs. As it has been shown in Section 3 of Ref. [25] the potential current of the range B, $j^a_{\mu,\,B}(2,p.c.)$, of the weak vector nuclear MECs satisfies the nuclear Conserved Vector Current (CVC) equation

$$q_\mu j^a_{\mu,\,B}(2,p.c.) \,=\, [V_B,\, j^a_0(1)]\,, \tag{1.5}$$

where V_B is the OBEP of the same range B and $j^a_0(1)$ is the one-body vector charge density. In its turn, an analogous current of the weak axial nuclear MECs, $j^a_{5\mu,\,B}(2,p.c.)$, fulfills (see Section 3.1 and Appendix A of Ref. [20]) the nuclear Partially Conserved Axial Current (PCAC) equation of the form

$$q_\mu j^a_{5\mu,\,B}(2,p.c.) \,=\, [V_B,\, j^a_{50}(1)] \,+\, if_\pi m^2_\pi \Delta^\pi_F(q^2)\mathcal{M}^a_B(2)\,, \tag{1.6}$$

where $j^a_{50}(1)$ is the one-body axial charge density and $\mathcal{M}^a_B(2)$ is the associated two-nucleon pion absorption/production amplitude. Further, f_π is the pion decay constant and Δ^π_F is the pion propagator. So the potential current in this case depends only on the employed potential model. Our definition of the MEC operators guarantees that there is no double counting if the MEC effects are calculated with the wave functions from Eq. (1.2).

Let us note that the presented approach has already been successfully applied to the description of nuclear phenomena. For example, in Refs. [26, 27], the cross section of the reaction of the backward deuteron disintegration,

$$e \,+\, d \,\to\, e' \,+\, n \,+\, p\,, \tag{1.7}$$

is well described up to energies of 1 GeV [28, 29]. In the weak sector, the capture rate of the reaction

$$\mu^- \,+\, {}^3He \,\to\, \nu_\mu \,+\, {}^3H\,, \tag{1.8}$$

was calculated in Ref. [30]. The obtained value $\Gamma_0 = 1502 \pm 32\ \mathrm{s}^{-1}$ is in a very good agreement with the result of the precise experiment [31, 32], $\Gamma^{exp}_0 = 1494.0 \pm 4.0\ \mathrm{s}^{-1}$.

The procedure of implementing phenomenological strong form factors into the electromagnetic MECs of the OBE type was in detail discussed in conjunction with the Bethe-Salpeter equation in Ref. [33]. Similar procedure for the weak axial MECs was performed in Ref. [34]. The construction of our weak axial nuclear MECs in conjunction with the Schroedinger equation (1.2) stems in Ref. [20] from these currents. The set of Eqs. (1.5) guarantees the gauge invariance of the calculations with the weak vector current, whereas the set of Eqs. (1.6) provides the correct relation between the weak axial current and the related pion production/absorption amplitude. When one deals with potentials of particular form such as the OBEPs containing phenomenological form factors, the splitting of the continuity equation for the currents into the set of equations for particular exchanges follows naturally. On the other hand, in the given case and also for a potential V of general form one can generate conserved currents in the weak vector sector using the minimal substitution [35, 36]. For the weak axial sector, an effective procedure for constructing the MECs satisfying the nuclear form of the PCAC,

$$q_\mu j^a_{5\mu}(2,p.c.) \,=\, [V,\, j^a_{50}(1)] \,+\, if_\pi m^2_\pi \Delta^\pi_F(q^2)\mathcal{M}^a(2)\,, \tag{1.9}$$

is not known.

Since the most important weak axial MECs are the $\Delta(1232)$ excitation currents of the π and ρ ranges, we pay particular attention to them. At first we generalize them so that also pieces depending explicitly on the off-shell parameters Y and Z [37] are included. Besides, we construct the Δ excitation currents also from the 'gauge symmetric' Lagrangians [38–40]. In calculations of the capture rates, we take specific values of the parameters Y and Z, which allows us to study the model dependence. In particular, if we take Y and Z as obtained from the requirement that the $\pi N\Delta$ and $\rho N\Delta$ interactions should not change the number of degrees of freedom of the free Δ [41–43], we obtain the same capture rates as calculated with the Δ excitation currents constructed from the gauge symmetric Lagrangians.

In our hybrid calculations, we apply the weak MECs that were used in [13]. However, we require that the weak vector MECs satisfy Eq. (1.5) and that the leading order terms of the weak axial MECs satisfy Eq. (1.6), also for the Gaussian strong form factor. In order that the weak axial MECs satisfy the nuclear form of the PCAC (1.6), one needs a potential current constructed in [44]. In the weak vector sector, in addition to the pion pair and pion-in-flight terms, we employ the Δ excitation current of the π and ρ ranges. These vector MECs are well known from the study of the process $n + p \to d + \gamma$ at the threshold in the TAA [45]. Since the Δ excitation currents saturate about 30 % of the MEC effect, we find it necessary to include them here, too. Let us note that the strength of the Δ excitation current of the π range constructed within the TAA from the gauge symmetric $\pi N\Delta$ Lagrangian is close to the strength of such a current constructed within the heavy baryon EFT [46]. The inspection of Table 2 [46] also confirms that this current should contribute non-negligibly. As we shall see, our calculations confirm that it is really the case.

In numerical calculations of the doublet capture rate, we use in the hybrid approach the same value of the cutoff Λ_π as in the TAA, which allows us to extract from the comparison of the doublet capture rates a unique value of the LEC $\hat{d}^R$.

In Section II, we discuss briefly the methods and inputs necessary for the calculations, in Section III we display the equations for the capture rates and in Section IV, we present the results. We conclude in Section V. Further, in Appendix A, we present the weak currents of the TAA and then we collect the Fourier transforms of the used MECs. In Appendix B we consider analogously the EFT currents, and in Appendix C we deliver all the multipoles of the currents used in the numerical calculations of the capture rates.

II. METHODS AND INPUTS

To obtain the capture rates one must calculate first of all the matrix elements of the weak nuclear currents between the initial and final nuclear states. Here we describe the needed ingredients of these calculations.

A. Weak nuclear currents of the TAA

The weak hadron current, triggering the reaction (1.1), is

$$j^-_\mu \,=\, j^-_\mu \,+\, j^-_{5\mu}\,. \tag{2.1}$$

The weak vector and weak axial nuclear currents j^-_μ and $j^-_{5\mu}$, respectively, consist of the one- and two-nucleon parts presented in Appendix A. There is practically no uncertainty

associated with the one-body part. Hence we concentrate on the effects of the two-body currents in the channel $d \to {}^1S_0$.

The weak axial nuclear MEC $j^-_{5\mu}(2)$ that we consider here is of the OBE-type with the π and ρ exchanges. It can be divided [44] into the potential and non-potential currents. The potential current of the range B, $j^-_{5\mu,B}(2, p.c.)$, satisfies the nuclear PCAC equation (1.6).

The main part of the non-potential weak axial exchange currents contain the model independent ρ-π current and the Δ excitation currents that are model dependent. In our calculations, we shall adopt the $\pi N\Delta$ and $\rho N\Delta$ Lagrangians used for many years [37, 47] to study the πN reactions and the pion photo- and electroproduction on a nucleon (model I) and also the gauge symmetric Lagrangians proposed recently [38, 39]. In model I, we derived the Δ excitation MECs from Lagrangians possessing the hidden local $SU(2)_L \times SU(2)_R$ symmetry [40]. In particular, the vertices containing the Δ isobar field were chosen as

$$\mathcal{L}_{N\Delta\pi\rho a_1} = \mathcal{L}_{N\Delta\pi a_1} + \mathcal{L}^1_{N\Delta\rho} + \mathcal{L}^2_{N\Delta\rho}\,, \tag{2.2}$$

where

$$\mathcal{L}_{N\Delta\pi a_1} = \frac{f_{\pi N\Delta}}{m_\pi}\bar{\Psi}_\mu \vec{T}\mathcal{O}_{\mu\nu}(Z)\Psi \cdot (\partial_\nu \vec{\pi} + 2 f_\pi g_\rho \vec{a}_\nu) + h.\,c.\,, \tag{2.3}$$

$$\mathcal{L}^1_{N\Delta\rho} = -g_\rho \frac{G_1}{M}\bar{\Psi}_\mu \vec{T}\mathcal{O}_{\mu\eta}(Y)\gamma_5\gamma_\nu\Psi \cdot \vec{\rho}_{\eta\nu} + h.\,c.\,, \tag{2.4}$$

$$\mathcal{L}^2_{N\Delta\rho} = g_\rho \frac{G_2}{M^2}\bar{\Psi}_\mu \vec{T}\mathcal{O}_{\mu\eta}(X)(\partial_\nu\Psi) \cdot \vec{\rho}_{\eta\nu} + h.\,c.\,. \tag{2.5}$$

Here $\vec{T}$ is the operator of the isospin $1/2 \to 3/2$ transition. The constant $f_{\pi N\Delta}$ is extracted from the Δ isobar width [48] with the result

$$\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2 /4\pi = 0.767 \pm 0.024\,\mathrm{fm}^2\,. \tag{2.6}$$

Besides, $G_1 = 2.525$ [37]. Further, the operator $\mathcal{O}_{\mu\nu}(B)$ is taken in the form [37, 47, 49, 50]

$$\mathcal{O}_{\mu\nu}(B) = \delta_{\mu\nu} + C(B)\,\gamma_\mu\gamma_\nu\,, \tag{2.7}$$

$$C(B) = -\left(\frac{1}{2} + B\right)\,. \tag{2.8}$$

The parameters X, Y and Z do not influence the on–shell properties of the Δ isobar, hence they are called off-shell parameters. They were systematically extracted from the data in Refs. [37, 47, 49, 50].

On the other hand, the parameters entering the Lagrangians (2.2) were restricted in [41, 42] on the basis of field theoretical arguments by the requirement that they should conserve the same number of degrees of freedom of the Δ isobar as possessed by the free one. This resulted in that it should be

$$Y = 0\,, \quad Z = \frac{1}{2}\,, \quad G_2 = 0\,. \tag{2.9}$$

This was criticized in [37], because the requirement $G_2 = 0$ fixes the ratio of the multipole E2 to multipole M1 amplitudes kinematically, which is unacceptable and the restriction (2.9) was refused as a whole. In our opinion the consistency requirement $G_2 = 0$ means that the

model I can be only applied if the amplitudes derived from the Lagrangian $\mathcal{L}^2_{N\Delta\rho}$, Eq. (2.5), do not contribute.
Later on, the problems related to the Lagrangians (2.2) were reconsidered in [38, 39, 43]. In particular it was shown in [43] within the framework of the Hamiltonian formalism with constraints that the result $Z = \frac{1}{2}$ persists.
Since the contribution to the effects of the Δ excitation currents for the process (1.1) from the Lagrangian (2.5) is negligible, we consider in our calculations two sets of parameters Y and Z. In the set Ia we take

$$Y \,=\, Z \,=\, -\frac{1}{2}\,. \tag{2.10}$$

In its turn, the set Ib is defined as

$$Y \,=\, 0\,, \quad Z \,=\, \frac{1}{2}\,. \tag{2.11}$$

The Δ excitation currents corresponding to the set (2.10) were used in all previous calculations of the MECs effects for the reaction (1.1).
The $\pi N\Delta$ and $\rho N\Delta$ Lagrangians of the model II [38, 39] do not contain any off-shell parameters. They are constructed in such a way that they conserve the degrees of freedom of the free Δ isobar. We write them as [40]

$$\mathcal{L}^{g.s.}_{N\Delta\pi a_1} \,=\, \frac{f_{\pi N\Delta}}{m_\pi M_\Delta}\, \varepsilon_{\mu\nu\alpha\beta}\, [(\partial_\mu \bar{\Psi}_\nu)\, \vec{T}\gamma_5\gamma_\alpha\, \Psi] \cdot (\partial_\beta \vec{\pi} \,+\, 2f_\pi g_\rho \vec{a}_\beta) \,+\, h.c.\,, \tag{2.12}$$

$$\mathcal{L}^{g.s.}_{N\Delta\rho} \,=\, \frac{G_1}{M M_\Delta}\, g_\rho\, \Big\{ \varepsilon_{\mu\nu\alpha\beta}\, [(\partial_\mu \bar{\Psi}_\nu)\, \vec{T}\gamma_\alpha\gamma_\lambda\, \Psi] \,+\, [(\partial_\mu \bar{\Psi}_\beta - \partial_\beta \bar{\Psi}_\mu)\vec{T}\gamma_5\gamma_\mu\gamma_\lambda\, \Psi] \Big\} \cdot \vec{\rho}_{\lambda\beta} \,+\, h.c.\,. \tag{2.13}$$

The values of the coupling constants are obtained from the condition that the new Lagrangians, Eq. (2.12) and Eq. (2.13) and the standard Lagrangians, Eq. (2.3) and Eq. (2.4), respectively, are equivalent for the on–shell Δ isobar.
It turns out that the Δ excitation currents of this model differ from those of the model I, set Ia (2.10), only by the factor $(M/M_\Delta)^2 \,\approx\, 0.58$ [$M(M_\Delta)$ is the nucleon (Δ isobar) mass].

B. Weak nuclear currents of the EFT approach

For the estimation of the MECs effect in the hybrid calculations, we use the MECs operator of Ref. [13] from which we omit the second term at the right hand side of Eq. (19). This term is suppressed by the factor $1/M$ in comparison with the leading term, which provides by itself only a small contribution to the weak axial MECs effect. In the weak vector sector, Ando *et al.* [13] present in Eq. (17) the non-relativistic version of the currents $J^\mu_{em}(a)$ and $J^\mu_{em}(b)$ of the set (C.4), constructed within the heavy baryon EFT [46]. They coincide with our π-pair term (A9) and pion-in-flight term (A10), constructed within the formalism of hard pion Lagrangians. Here we add to them the Δ excitation currents of the π and ρ ranges (A11) and (A13). It can be shown that the Δ excitation current of the π range (A11), if taken with the set (2.10) and multiplied by the factor $(M/M_\Delta)^2$ corresponds to the Δ excitation current $J^\mu_{em}(f)$, Eq. (C.4) of Ref. [46]. The TAA calculations show that the currents (A11) and (A13) contribute non-negligibly. So in order to assess the capture rate $\Lambda_{1/2}(d \,\rightarrow\, {}^1S_0)$ correctly, one should include explicitly also these currents in the EFT

calculation of the capture rates. In our opinion the current (A13) can be obtained from the heavy-fermion Lagrangians of Appendix C [46] as well.
In the weak axial sector, we add to the weak axial MECs [13] the π potential term (A17). This ensures that also the weak axial MECs of the EFT approach satisfy in the leading order the PCAC constraint (1.6) [44].
The weak axial MECs of the heavy baryon EFT contain the known LECs $\hat{c}_i$, ($i = 1, 2, 3, 4$), c_6 and the unknown LECs $\hat{d}_1$ and $\hat{d}_2$. These two constants enter in the calculations of the observables effectively in the combination $\hat{d}_1 + 2\hat{d}_2$ or as used in the formalism of Ref. [13] in the combination

$$\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}\,. \tag{2.14}$$

On the other hand the combination $\hat{d}_1 + 2\hat{d}_2$ is also expressed in terms of the LEC c_D as [51]

$$\hat{d}_1 + 2\hat{d}_2 = -\frac{M_N}{\Lambda_\chi g_A} c_D\,, \tag{2.15}$$

where M_N=0.939 GeV is the nucleon mass and $\Lambda_\chi \approx 1$ GeV.
Let us note that the constant $\hat{d}^R$ (c_D) enters not only the short range part of the weak axial MEC but also the pion production/absorption amplitude in the NN collisions and the NNN force.
The dependence of the LECs on the off-shell parameter Z was discussed by Bernard *et al.* [52] in the model where the LECs are saturated by the heavy mesons and the Δ resonance. In parallel with the model Ia (2.10), in which $Z = -\frac{1}{2}$ we obtain the set IIa

$$\hat{c}_2 = 3.37\,, \quad \hat{c}_3 = -4.70\,, \quad \hat{c}_4 = 3.31\,, \quad (\mathrm{IIa})\,, \tag{2.16}$$

whereas for the case Ib (2.11) in which $Z = \frac{1}{2}$ the LECs of the model IIb are

$$\hat{c}_2 = 1.98\,, \quad \hat{c}_3 = -3.31\,, \quad \hat{c}_4 = 2.61\,, \quad (\mathrm{IIb})\,. \tag{2.17}$$

Another set of these constants, which we design as the set IIc, has been extracted in [53] from the data,

$$\hat{c}_2 = 1.67 \pm 0.09\,, \quad \hat{c}_3 = -3.66 \pm 0.08\,, \quad \hat{c}_4 = 2.11 \pm 0.08\,, \quad (\mathrm{IIc})\,. \tag{2.18}$$

If one omits from the LECs $\hat{c}_2$, $\hat{c}_3$ and $\hat{c}_4$ the contribution from the Δ isobar, one gets the model IId

$$\hat{c}_2 = 0.047\,, \quad \hat{c}_3 = -1.371\,, \quad \hat{c}_4 = 1.63\,, \quad (\mathrm{IId})\,. \tag{2.19}$$

For the constants $\hat{c}_1$ and c_6 we take [13, 53]

$$\hat{c}_1 = -0.60 \pm 0.13\,, \quad c_6 = 3.70\,. \tag{2.20}$$

New set of the LECs has recently been delivered in Ref. [54]. We take it as model IIe,

$$\hat{c}_1 = -0.85\begin{array}{c}+0.2\\-0.5\end{array},\quad \hat{c}_2 = 3.1 \pm 0.2\,, \quad \hat{c}_3 = -4.4\begin{array}{c}+1.2\\-1.0\end{array},\quad \hat{c}_4 = 3.3\begin{array}{c}+0.5\\-0.2\end{array},\quad (\mathrm{IIe})\,. \tag{2.21}$$

These new LECs are close to the set IIa (2.16) obtained for $Z = -\frac{1}{2}$. However, these new LECs are extracted with much larger errors than the set IIc (2.18).

C. Nuclear potentials

We use the Nijmegen I (NI) and Nijmegen 93 (N93) [24] OBEPs. The couplings and cutoffs, entering these potentials, are employed also in the MECs. In particular, the cutoffs are

$$\Lambda_\pi = 827.5\,(\mathrm{NI})\,, \quad \Lambda_\pi = 1177.11\,(\mathrm{N93})\,. \tag{2.22}$$

Let us note that our main results are related to the high quality second generation NI potential with χ^2/N_{data}=1.03, whereas for the N93 potential χ^2/N_{data}=1.87.

III. CAPTURE RATES

The capture rates $\Lambda_{1/2}$ and $\Lambda_{3/2}$ are related to the statistical capture rate, Λ_{stat}, as

$$\Lambda_{stat} \;=\; \frac{1}{3}\,\Lambda_{1/2} \,+\, \frac{2}{3}\,\Lambda_{3/2}\,. \tag{3.1}$$

Using the method of Ref. [55], we write the statistical capture rate for the process (1.1) in terms of the multipoles,

$$\begin{aligned}\Lambda_{stat} \;&=\; \frac{M_n}{3}\left[\frac{G_F\cos\theta_C\phi_\mu(0)}{\pi}\right]^2 \int_0^{\nu_{max}} d\nu\,(\nu^2/\kappa_0) \sum_{\lambda j_f,\,J}\Big\{|<\lambda j_f||i\hat{T}^{el}_J-\hat{T}^{mag}_J||d>|^2 \\ &\quad +|<\lambda j_f||\hat{L}_J-\hat{M}_J||d>|^2\Big\}\,,\end{aligned} \tag{3.2}$$

where

$$\begin{aligned}\kappa_0 \;&=\; \sqrt{M_n(\Delta-\nu-\nu^2/4M_n)}\,, \quad \nu_{max}=2M_n(-1+\sqrt{1+\Delta/M_n})\,, \\ \Delta \;&=\; m_\mu+m_d-2M_n-|\epsilon_\mu|\,,\end{aligned} \tag{3.3}$$

$M_n\,(m_d)$ is the neutron (deuteron) mass, and $\epsilon_\mu = -0.00267$ MeV is the binding energy of the muon. Further in (3.2), the weak interaction constant $G_F = 1.16637\times10^{-5}$ GeV^{-2} [48], $\cos\theta_C$=0.9749 and the wave function of the bound muon at the center of the deuteron is $\phi_\mu(0) = (m_{\mu,r}\,\alpha)^{3/2}/\sqrt{\pi}$, $m_{\mu,r}$ is the reduced mass of the muon and α is the fine structure constant.
The multipoles and their reduced matrix elements are defined in Section 4.1 of Ref. [20].
Generally, the capture rates Λ_F for the hyperfine state F can be obtained from the equation [55],

$$\Lambda_F \;=\; \Lambda_{stat} \,+\, C_F\,\delta\Lambda\,, \tag{3.4}$$

where

$$C_F \;=\; (-1)^{F+\frac{1}{2}}\left\{\begin{array}{ccc} J_i & \frac{1}{2} & F \\ \frac{1}{2} & J_i & 1\end{array}\right\}\,, \tag{3.5}$$

and $\delta\Lambda$ depends on the nuclear dynamics. Here J_i is the total angular momentum of the initial nucleus.
For the reaction (1.1), J_i=1 and

$$C_{\frac{1}{2}} \;=\; \frac{1}{3}\,, \quad C_{\frac{3}{2}} \;=\; -\frac{1}{6}\,, \tag{3.6}$$

whereas

$$\delta\Lambda \;=\; \sqrt{6}M_n\left[\frac{G_F\cos\theta_C\phi_\mu(0)}{\pi}\right]^2\int_0^{\nu_{max}} d\nu\,(\nu^2/\kappa_0)\sum_{\lambda j_f,\,J\,J'}\hat{J}\,\hat{J}'\,(-1)^{j_f}\left\{\begin{array}{ccc} J_i & J & j_f \\ J' & J_i & 1\end{array}\right\}$$

$$\Big\{ i^{J-J'}\begin{pmatrix} J & J' & 1 \\ -1 & 1 & 0\end{pmatrix} < \lambda j_f||i\hat{T}^{el}_J - \hat{T}^{mag}_J||d >< \lambda j_f||i\hat{T}^{el}_{J'} - \hat{T}^{mag}_{J'}||d >^*$$

$$-2\sqrt{2}\begin{pmatrix} J & J' & 1 \\ -1 & 0 & 1\end{pmatrix}\Re\Big[i^{J-J'} < \lambda j_f||i\hat{T}^{el}_J - \hat{T}^{mag}_J||d >< \lambda j_f||\hat{L}_{J'} - \hat{M}_{J'}||d >^* \Big]$$

$$+\, i^{J-J'}\begin{pmatrix} J & J' & 1 \\ 0 & 0 & 0\end{pmatrix} < \lambda j_f||\hat{L}_J - \hat{M}_J||d >< \lambda j_f||\hat{L}_{J'} - \hat{M}_{J'}||d >^* \Big\}\,. \qquad (3.7)$$

Here $\hat{J} = \sqrt{2J+1}$ and the symbols $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3\end{pmatrix}$ and $\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6\end{array}\right\}$ are Wigner's $3jm$ and $6j$ symbols, respectively [56].

IV. RESULTS AND DISCUSSION

Here we present first the results for the capture rates obtained for the reaction (1.1) in the formalism of the TAA and then in the hybrid calculations.

A. Results for the TAA

In the formalism of the TAA, we calculated the contributions to the capture rates from all channels $d \rightarrow {}^{2S+1}L_{j_f}$, where L=S,P,D,F, j_f=0,1,2 and from the multipoles J=0,1,2,3 of the one-nucleon currents. The contribution of the weak MECs was taken into account in the multipole J=1 and in the channel $d \rightarrow {}^1S_0$. We also estimated the MEC effect due to the weak vector MECs $\vec{j}^{\,-}(p.t.)$, Eq. (A9), and $\vec{j}^{\,-}(\pi\pi)$, Eq. (A10) in the channels $d \rightarrow {}^3P_{0,1,2}$. The results of the calculations of the doublet capture rate $\Lambda_{1/2}$ for the reaction (1.1) are presented in Table I.

It is seen from Table I that the results depend on the potential and current model used. As noted above, our basic model is NI/Ib: the potential NI qualitatively supersedes the N93 one and the current model Ib should be preferred because the parameters of the Δ excitation currents are restricted by the reasonable demand that the number of degrees of freedom of the Δ isobar should be conserved. Moreover, the same results as displayed in the third row of Table I are obtained with the potential NI and with the current model based on the Lagrangians (2.3) and (2.4). As is seen from the fifth column of the Table I, the resulting MECs effect for this case is 16.7 s^{-1}, which is $\approx$ 4 %. The estimated error in $\Lambda_{1/2}$ for this model due to the 3 % variation of the constant $\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2/4\pi$ given in Eq. (2.6) is $\approx$ 1 s^{-1}, which is $\approx$ 0.25 %.

Let us note that the time component of the weak axial MECs contributes by +2.3 (+2.9) s^{-1} for the models NI/Ia and NI/Ib (N93/Ia and N93/Ib). Our calculations show that the effect of the weak vector Δ excitation currents is $\approx 4-6$ s^{-1}. It is a non-negligible contribution to $\Lambda_{1/2}$ of the order of $1-1.5$ %, which cannot be neglected if the expected error of the data is $\approx$ 1.5 %. If one looks for the effect of the heavy meson exchanges, one obtains for the

TABLE I: Partial contributions to $\Lambda_{1/2}$ (in s^{-1}). The value of the constant $\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2/4\pi = 0.783$ fm^2 is used. In the first column, the potential and the current model are displayed. In the second column, the contribution from the one-body current in the channel $d \to {}^1S_0$ is given. In the third column, the contribution of the MECs in the channel $d \to {}^1S_0$ is presented, whereas in the fourth column, the contribution of the one-body current to all considered channels but $d \to {}^1S_0$ is given. In the fifth column, we give the total MECs effect and in the last column, all the contributions are summed up.

	IA_0	$\Delta\mathrm{MEC}_0$	ΔIA	MEC	IA+MEC
NI/Ia	239.2	22.0	160.4	23.8	423.4
NI/Ib	239.2	14.8	160.4	16.7	416.3
N93/Ia	238.8	29.1	160.4	30.8	430.0
N93/Ib	238.8	22.0	160.4	23.8	423.0

model NI/Ib that

$$\Delta\Lambda_{1/2}^{\mathrm{MEC}} = 24.3\,\mathrm{s}^{-1} - 7.6\,\mathrm{s}^{-1} = 16.7\,\mathrm{s}^{-1}\,, \tag{4.1}$$

where the first number at the right-hand side of the equation is due to the MECs of the pion range and the second number stems from the heavy meson exchanges. So the effect of the short range MECs is ≈ 30 % of the long range one.
In Table II we present the contribution from particular multipoles to the capture rates calculated with the IA currents. It is seen that the contributions from higher multipoles not taken into account cannot change the results much.

TABLE II: Partial contributions to the capture rates (in s^{-1}) from the multipoles J, calculated in the IA approximation. The neutron-neutron ${}^{2S+1}L_{j_f}$ partial waves with L=0,1,2,3 and j_f=0,1,2 are taken into account. In the second column, only the contribution from the channel $d \to^1 S_0$ to the multipole J=1 is considered. In the third column, the capture rates are calculated with the multipoles J=0 added; in the fourth column, all the multipoles with J=1 but arising from the channel $d \to^1 S_0$ are added; in the fifth (sixth) column, the multipoles J=2 (J=3) are added.

J	1 (${}^1S_0$)	0	1	2	3
Λ_{stat}	83.2	93.2	119.6	139.4	141.0
$\Lambda_{1/2}$	239.2	249.3	322.4	394.6	399.6
$\Lambda_{3/2}$	5.1	15.2	18.3	11.9	11.7

We now give for the model NI/Ib the final results for the capture rates. In the channel $d \to {}^1S_0$,

$$\Lambda^0_{stat} = 88.1\,\mathrm{s}^{-1}\,, \quad \Lambda^0_{1/2} = 254.0\,\mathrm{s}^{-1}\,, \quad \Lambda^0_{3/2} = 5.2\,\mathrm{s}^{-1}\,. \tag{4.2}$$

The full calculations provide

$$\Lambda_{stat} = 146.4\,\mathrm{s}^{-1}\,, \quad \Lambda_{1/2} = 416.3\,\mathrm{s}^{-1}\,, \quad \Lambda_{3/2} = 11.4\,\mathrm{s}^{-1}\,. \tag{4.3}$$

The result $\Lambda_{1/2} = 416.3\,\mathrm{s}^{-1}$, Eq. (4.3), seems to be in agreement with the one of Eq. (38) of Ref. [6], $\Lambda_{1/2} = 416 \pm 7\,\mathrm{s}^{-1}$. However, it is more correct to compare this result with the value $\Lambda_{1/2} = 430\ \mathrm{s}^{-1}$ of our Table I, obtained from the first generation realistic potential N93 and the TAA current model Ia, because this sort of potentials and of the current model was used also in [6]. In Ref. [7], the value $\Lambda_{1/2} \approx 400\,\mathrm{s}^{-1}$ was reported, as the result of calculations with similar potentials and currents as in [6]. On the other hand, according to Ref. [13], the contribution of the higher partial waves was later reevaluated [7] and an enhancement of the $\Lambda_{1/2}$ by $\approx 10\ \mathrm{s}^{-1}$ was achieved. Then $\Lambda_{1/2} \approx 410\ \mathrm{s}^{-1}$ is in good agreement with the result of [6], but it is by 5 % smaller than our corresponding value of $\Lambda_{1/2} = 430\mathrm{s}^{-1}$. Let us note the calculations of Ref. [57] reporting $\Lambda_{1/2} = 402\,\mathrm{s}^{-1}$, also performed with the first generation realistic potentials and the current model Ia.

B. Results for the EFT currents

Here we provide the results of calculations of the LECs $\hat{d}^R$ and c_D by comparing the doublet transition rate $\Lambda^0_{1/2}$ calculated with the weak MECs of Section IV A. We made the calculations with the potential NI and the weak MECs discussed in Section II B. Using the Goldberger-Treiman relation we connected the coupling of MECs [13] with the constant $g_{\pi NN}$ that we took from the potential. Since the cutoff Λ_π is also taken from the potential, our hybrid calculations are consistent as much as possible. As in Ref. [13], we use here the weak form factors in the linear approximation in the expansion in the four momentum transfer q^2, given in Eqs. (B1), and (B2). However, this provides only a small difference in the results, in comparison with the full q^2 dependence.
It follows from Table I for the model NI/Ia that $\Lambda^0_{1/2} = 261.2\ \mathrm{s}^{-1}$. This provides for the model NI/IIa the value of the LEC $\hat{d}^R = 3.225$. In both models, $Z = -\frac{1}{2}$. For the other considered models, we take $\Lambda^0_{1/2} = 254.0\ \mathrm{s}^{-1}$ obtained for the model NI/Ib and the resulting constants $\hat{d}^R$ and c_D are presented in Table III.

TABLE III: Values of the LECs $\hat{d}^R$ and c_D obtained by comparing the doublet capture rate for the channel $d \to {}^1S_0$, calculated in Section IV A for the model NI/Ib, and the one calculated with the weak MECs of Section II B. The value $Z = \frac{1}{2}$ is taken in the model NI/Ib and also in the model NI/IIb, in calculating the contribution of the Δ resonance to the LECs $\hat{c}_i$, $i = 2, 3, 4$. In the model NI/IId, the contribution of the Δ resonance to the LECs $\hat{c}_i$, $i = 2, 3, 4$ is omitted, whereas the Δ excitation current of the pion range is explicitly taken into account.

	NI/IIb	NI/IIc	NI/IId	NI/IIe
$\hat{d}^R$	2.410	2.155	0.625	2.680
c_D	2.173	2.436	-0.231	2.407

Comparing the second and the fourth columns of Table III shows that taking into account the Δ resonance effect by the method of the resonance saturation of the LECs and taking it into account explicitly by calculating the MECs effect is not equivalent. Also comparing the third and the last columns one finds about 20 % change in the value of the constant $\hat{d}^R$. Having in mind the large uncertainty in the LECs of the set IIe (2.21) one concludes

that the change is not essential. Let us also note that the value $\hat{d}^R$=2.68 was obtained with $Y = Z = -\frac{1}{2}$ in the Δ excitation currents. If one employs $Y = 0$ and $Z = -\frac{1}{2}$, the value of $\hat{d}^R$=2.66, so it changes insignificantly. The value of the Λ_π entering the NI potential (2.22) is close to the value of one of the cutoffs, Λ=800 MeV, used in the analysis of the triton beta decay [18]. For this value of the cutoff, the extracted $\hat{d}^R = 3.90 \pm 0.10$ [18], which is enhanced at least by 30 % in comparison with $\hat{d}^R$ from our Table III.

We have also calculated the influence of the weak axial potential current $\vec{j}^{\,a}_{5\mu,\pi}(2, p.c.)$ on the value of the constant $\hat{d}^R$. As discussed in Ref. [44], this current is usually absent in calculations of the weak processes. If we omit it the value of the constant $\hat{d}^R$=2.410 in the second column of Table III increases to $\hat{d}^R$=2.740, thus it changes by $\approx$ 13 %. It follows that if one would like to extract the value of $\hat{d}^R$ with an accuracy better than 10 %, then one should take the contribution of the potential current into account. The calculations also show that omitting the potential current causes an enhancement of the doublet transition rate $\Lambda_{1/2}$ by $\approx$ 1 %.

The constant c_D has recently been extracted [51], together with another LEC c_E, entering the contact part of the NNN force, from the data on the triton beta decay, binding energies and point-proton radii of the 3N system and ^{4}He nucleus, with resulting value c_D=-0.2. As it is seen from our Table III, only the value c_D=-0.231, corresponding to the model NI/IId, is in agreement with the analysis of Ref. [51]. Let us note that with the choice $\hat{c}_4 = -\hat{c}_3$=3.4 [51] one obtains the value $\hat{d}^R$=1.1 using c_D=-0.2.

It follows from our calculations that the effect of the time component of the weak axial MECs is $\approx$ -1 s^{-1}, which is in agreement with Ref. [13]. This is in contrast to the TAA calculations based on the hard pion Lagrangians, where the time component contributes as $\approx$ +2 s^{-1}. The short range part of the hard pion time component reduces the value $\hat{d}^R$= 2.155 (see the third column of Table III) to $\hat{d}^R$= 1.91, if one uses this component instead of the soft pion one in fitting $\hat{d}^R$.

Our $\Lambda^0_{1/2} = 254.0$ s^{-1} differs by 9 s^{-1} from the same quantity, $\Gamma^{L=0}_{\mu d} = 245$ s^{-1}, given in Table 1 of Ref. [13]. The main part of the difference can be assigned to the difference of $\approx$ 7 s^{-1} in the contribution from the one-body currents, obtained from comparing IA$_0$=239.2 s^{-1} (see our Table I) with the value of $\Gamma_{\mu d}$=232 s^{-1} of Table 2 [13]. On the other hand, our MECs effects, ΔMEC$_0$=14.8 s^{-1}, are close to 13 s^{-1} obtained in [13].

V. CONCLUSIONS

We have evaluated the capture rates for the reaction of muon capture in deuterium (1.1), both using the TAA currents and those derived within the EFT approach.

The weak TAA currents, presented in Appendix A, are of the one-boson exchange type obtained from the hard pion chiral Lagrangians and they satisfy the nuclear CVC and PCAC constraints, Eqs. (1.5) and (1.6), respectively. The final state neutron-neutron wave functions were generated from the high quality second generation potential NI and from the realistic potential N93 [24]. Since the potentials are also of the one-boson exchange type, employing in the TAA currents the same couplings and strong form factors (1.4), we performed fully consistent calculations, presented in Section IV A. For the main object of interest, the doublet capture rate $\Lambda^0_{1/2}$ for the channel $d \rightarrow ^1 S_0$, we predict (see Table I,

model NI/Ib)

$$\Lambda^0_{1/2} \,=\, 254\,\pm\,3\,\mathrm{s}^{-1}\,. \tag{5.1}$$

This result was obtained with the neutron-neutron wave functions derived from the NI potential. The error reflects the uncertainty in the $\pi N\Delta$ and $\rho N\Delta$ couplings, possible effects of the neglected short range effects and applied approximations. In the model NI/Ib, the $\pi N\Delta$ and $\rho N\Delta$ couplings preserve the physical degrees of freedom of the free Δ isobar. The IA currents contribute to the value (5.1) of $\Lambda^0_{1/2}$ by 239 s^{-1}, whereas the MEC effect is 15 s^{-1}, which is $\approx$ 6 %. In the full calculations, we considered the neutron-neutron ${}^{2S+1}L_{j_f}$ partial waves with L=0,1,2,3 and j_f=0,1,2, and the contributions of the IA currents to the multipoles J=0,1,2,3. For the full doublet capture rate we got

$$\Lambda_{1/2} \,=\, 416\,\pm\,6\,\mathrm{s}^{-1}\,. \tag{5.2}$$

In addition, the estimated error includes also the uncertainty due to the neglect of the contribution from the higher multipoles.

The EFT currents that we used are discussed in Appendix B. The hybrid calculations of the capture rates accomplished with these currents are presented in Section IV B. These calculations are consistent to the extent that we again use in the MECs the couplings and strong form factors from the potential NI. We extract the unknown LEC $\hat{d}^R$ by comparing the capture rate $\Lambda^0_{1/2}$ with its numerical value calculated with the TAA currents. Besides, we adopt various available sets of the known LECs $\hat{c}_i$, discussed in Section II B. As is seen from Table III, the value of $\hat{d}^R$ changes within 25 % for various sets of $\hat{c}_i$. The exception is provided by the case in which the contribution of the Δ is eliminated from $\hat{c}_i$, and the Δ excitation current is taken into account explicitly. Then the value of $\hat{d}^R$ is suppressed by the factor $\approx$ 4.

Comparing our results with those of Ref. [13] we see that our calculations provide the value of $\Lambda^0_{1/2} = 254\,\mathrm{s}^{-1}$ which is by $\approx$ 4 % larger than the analogous value $\Gamma^{L=0}_{\mu d} = 245\,\mathrm{s}^{-1}$ of [13]. Equally, our total capture rate $\Lambda_{1/2} \,= 416\,\mathrm{s}^{-1}$ differs from $\Gamma_{\mu d} = 386\,\mathrm{s}^{-1}$ [13] by $\approx$ 7 %.

In conclusion we stress that the planned precise experimental investigation [10] of the reaction (1.1) is of fundamental importance. It will stimulate efforts to understand better the details and limits of application of both the TA and EFT approaches and will certainly shed more light on the value of the important LEC $\hat{d}^R$ (c_D).

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[1] M. Kirchbach, E. Truhlík, Sov. J. Part. Nucl. **17** (1986) 93 .
[2] D.F. Measday, Phys. Rep. **354** (2001) 243 .
[3] E. Ivanov, E. Truhlík, Nucl. Phys. **A316** (1979) 437 .
[4] E. Ivanov, E. Truhlík, Nucl. Phys. **A316** (1979) 451 .

[5] G.E. Dogotar, R.A. Eramzhyan, E. Truhlík, Nucl. Phys. **A326** (1979) 225 .
[6] J. Adam Jr. E. Truhlík, S. Ciechanowicz, K.M. Schmitt, Nucl. Phys. **A507** (1990) 675 .
[7] N. Tatara, Y. Kohyama, K. Kubodera, Phys. Rev. C**42** (1990) 1694 .
[8] M. Cargnelli, PhD thesis, Technical University of Vienna, 1987; W.H. Breunlich, M. Cargnelli, H. Fuhrmann, P. Kammel, J. Marton, J. Werner, J. Zmeskal, C. Petitjean, in Workshop on fundamental μ-physics, Los Alamos, 1986, LA10714C.
[9] J. Martino, Czech. J. Phys. **B36** (1986) 368 .
[10] MuSun Collaboration, Muon Capture on the Deuteron, http://www.npl.uiuc.edu/exp/musun.
[11] T. Ericson, W. Weise, Pions and Nuclei, Clarendon Press, Oxford, 1988.
[12] J. Carlson, R. Schiavilla, Rev. Mod. Phys. **70** (1998) 743 .
[13] S. Ando, T.S. Park, K. Kubodera, F. Myhrer, Phys. Lett. **B533** (2002) 25 .
[14] S. Weinberg, Physica **96 A** (1979) 327.
[15] J. Gasser, H. Leutwyler, Ann. Phys. (N.Y.) **158** (1984) 142 .
[16] J.W. Chen, T. Inoue, X. Ji, Y. Li, Phys. Rev. C **72** (2005) 061001(R).
[17] R.B.Wiringa, V.G.J.Stoks, R.Schiavilla, Phys. Rev. C**51** (1995) 38 .
[18] T.-S. Park, L.E. Marcucci, R. Schiavilla, M. Viviani, A. Kievsky, S. Rosati, K. Kubodera, D.-P. Min, M. Rho, Phys. Rev. C**67** (2003) 055206 .
[19] E. Truhlík, F.C. Khanna, Int. J. Mod. Phys. **A10** (1995) 499 .
[20] B. Mosconi, P. Ricci, E. Truhlík, Nucl. Phys. **A772** (2006) 81 .
[21] R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149** (1987) 1 .
[22] R. Machleidt, Adv. Nucl. Phys. **19** (1989) 189 .
[23] R. Machleidt, Phys. Rev. C**63** (2001) 024001 .
[24] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, J.J. de Swart, Phys. Rev. C**49** (1994) 2950 .
[25] J. Adam, Jr. , E. Truhlík, D. Adamová, Nucl. Phys. **A492** (1989) 556 .
[26] E. Truhlik, K.-M. Schmitt, Few–Body Systems **11** (1992) 155 .
[27] J. Smejkal, E. Truhlik, H. Goeller, Nucl. Phys. **A624** (1997) 655 .
[28] I. Sick, Prog. Part. Nucl. Phys. **47** (2001) 245 .
[29] R. Gilman, F. Gross, J. Phys. **G28** (2002) R37 .
[30] J. Congleton, E. Truhlik, Phys. Rev. C**53** (1996) 956 .
[31] A.A. Vorobyov *et al.* Hyperfine Interactions **101/102** (1996) 413 .
[32] P. Ackerbauer *et al.* Phys. Lett. **B417** (1998) 224 .
[33] F. Gross, D.O. Riska, Phys. Rev. C**36** (1987) 1928 .
[34] F.C. Khanna, E. Truhlik, Nucl. Phys. **A673** (2000) 455 .
[35] A. Buchmann, W. Leidemann, H. Arenhoevel, Nucl. Phys. **A443** (1985) 726 .
[36] L.E. Marcucci, M. Viviani, R. Schiavilla, A. Kievsky, S. Rosati, Phys. Rev. C**72** (2005) 014001 .
[37] M. Benmerrouche, R.M. Davidson, N.C. Mukhopadhyay, Phys. Rev. C**39** (1989) 2339 .
[38] V. Pascalutsa, R. Timmermans, Phys. Rev. **C 60**, 042201(R) (1999).
[39] V. Pascalutsa, Phys. Lett. **B503** (2001) 85 .
[40] J. Smejkal, E. Truhlík, Phys. Rev. C**72** (2005) 015501 .
[41] L.M. Nath, B. Etamadi, J.D. Kimel, Phys. Rev. D**3** (1971) 2153 .
[42] L.M. Nath, B.K. Bhattacharyya, Z. Phys. **C5** (1980) 9 .
[43] V. Pascalutsa, Hadronic J. Suppl. **16** (2001) 1 .
[44] B. Mosconi, P. Ricci, E. Truhlík, Eur. Phys. J.**A25** (2005) 283 .
[45] J.F. Mathiot, Phys. Rep. **173** (1989) 63 .

[46] T.S. Park, D.P. Min, M. Rho, Nucl. Phys. **A596** (1996) 515 .
[47] M.G. Olsson, E.T. Osypowski, Nucl. Phys. **B87** (1975) 399 .
[48] C. Amsler *et al.*, Phys. Lett. **B667** (2008) 1 .
[49] R.M. Davidson, N.C. Mukhopadhyay, R.S. Wittman, Phys. Rev. Lett. **56** (1986) 804 .
[50] R.M. Davidson, N.C. Mukhopadhyay, R.S. Wittman, Phys. Rev. D**43** (1991) 71 .
[51] D. Gazit, S. Quaglioni, P. Navratil, Phys. Rev. Lett. **103** (2009) 102502 .
[52] V. Bernard, N. Kaiser, U.G. Meissner, Int. J. Mod. Phys. **E4** (1995) 193 .
[53] V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. **B475** (1995) 147 .
[54] V. Bernard, Prog. Part. Nucl. Phys. **60** (2008) 82 .
[55] J.D. Walecka, Nucl. Phys. **A258** (1976) 397 .
[56] D.A. Varshalovich, A.N. Moskalev, V.K. Khersonsky, Quantum Theory of Angular Momentum, World Scientific, Singapore, 1988.
[57] M. Doi, T. Sato, H. Ohtsubo, M. Morita, Nucl. Phys. **A511** (1990) 507 .
[58] M. Abramowicz, I.A. Stegun (Eds.), Handbook of Mathematical Functions, Applied Mathematics Series, vol. 55, National Bureau of Standards, 1964.
[59] P. Kammel, Muon capture and muon lifetime, talk at the workshop Fundamental Symmetries and Weak Interactions, Institute for Nuclear Theory, University of Washington, Sept.30–Dec.6, 2002.
[60] A. Liesenfeld *et al.*: Phys. Lett. **B468** (1999) 20 .

Appendix A: The weak currents of the TAA

The hadron currents consists of the one- and two-nucleon parts. The one-nucleon currents are of the form,

$$\vec{j}^{\,a} = \frac{1}{2M}\left[F_1^V(q^2)\,\vec{P} + iG_M^V(q^2)(\vec{\sigma}\times\vec{q})\right]\frac{\tau^a}{2}\,, \tag{A1}$$

$$\rho^{\,a} = F_1^V(q^2)\,\frac{\tau^a}{2}\,, \tag{A2}$$

$$\begin{aligned}\vec{j}_5^{\,a} = &\left\{g_A F_A(q^2)\left[\vec{\sigma} - \frac{1}{8M^2}\left[\vec{P}^{\,2}\vec{\sigma} - (\vec{\sigma}\cdot\vec{P})\vec{P} + (\vec{\sigma}\cdot\vec{q})\vec{q} - i(\vec{P}\times\vec{q})\right]\right]\right. \\ &\left. - \frac{g_P}{2Mm_\mu}\,(\vec{\sigma}\cdot\vec{q})\,\vec{q}\right\}\frac{\tau^a}{2}\,,\end{aligned} \tag{A3}$$

$$\rho_5^{\,a} = \left[\frac{g_A F_A(q^2)}{2M}(\vec{\sigma}\cdot\vec{P}) - \frac{g_P}{2Mm_\mu}(\vec{\sigma}\cdot\vec{q})\,q_0\right]\frac{\tau^a}{2}\,. \tag{A4}$$

Here $\vec{P} = \vec{p}^{\,\prime} + \vec{p}$, $q_\mu = p'_\mu - p_\mu$, where p'_μ (p_μ) is the four-momentum of the nucleon in the final (initial) state and the induced pseudoscalar form factor is

$$g_P(\vec{q}^{\,2}) = 2Mg_Am_\mu\Delta_F^\pi(\vec{q}^{\,2})\,. \tag{A5}$$

For the other weak form factors we employ the dipole parametrization,

$$F_1^V(q^2) = 1/(1 + q^2/M_V^2)\,, \quad M_V^2 = 0.711\ \mathrm{GeV}^2\,, \tag{A6}$$

$$F_A(q^2) = 1/(1 + q^2/M_A^2)\,, \quad M_A^2 = 1.04\ \mathrm{GeV}^2\,, \tag{A7}$$

We use for the constant g_A the value [48]

$$g_A = -1.2694 \pm 0.0028\,. \tag{A8}$$

1. The weak exchange currents

The two-nucleon part also consists of the weak vector and weak axial vector parts. We present first the weak vector MECs. They are

1. The π-pair term,

$$\vec{j}^{\,a}(p.t.) = -\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 F_1^V(q^2)\,\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}^2(\vec{q}_2^{\,2})\,\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{q}_2)\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a + (1\leftrightarrow 2)\,. \tag{A9}$$

2. The pion-in-flight term,

$$\begin{aligned}\vec{j}^{\,a}(\pi\pi) = &\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 F_1^V(q^2)\,\vec{q}_1\,(\vec{\sigma}_1\cdot\vec{q}_1)\,(\vec{\sigma}_2\cdot\vec{q}_2)\,\frac{1}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}\left[\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}^2(\vec{q}_2^{\,2})\right. \\ &\left. - \Delta_F^\pi(\vec{q}_1^{\,2})\,F_{\pi NN}^2(\vec{q}_1^{\,2})\right]\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a + (1\leftrightarrow 2)\,.\end{aligned} \tag{A10}$$

3. The Δ excitation current of the π range,

$$\vec{j}_\pi^{\,a}(\Delta) \;=\; -i\,\frac{q\,C_\pi^V}{9(M_\Delta - M)}\,F_1^V(q^2)\,\hat{q}\,\times\,\Big\{4\Big[1\,+\,f(Y,Z)\Big]\,\vec{q}_2\,\tau_2^a$$
$$+\Big[1\,-\,2f(Y,Z)\Big]\,i(\vec{\sigma}_1\times\vec{q}_2)\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\}$$
$$\times\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}^2(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2)\,+\,(1\,\leftrightarrow\,2)\,, \tag{A11}$$

where $\hat{q}\,=\,\vec{q}/|\vec{q}|$ and

$$f(Y,Z) \;=\; (1 - M/M_\Delta)\;[C(Y) + C(Z) + 2C(Y)C(Z)(2 + M/M_\Delta)]\;,$$
$$C(a) \;=\; -(\frac{1}{2} + a)\,,\quad C_\pi^V \;=\; 2G_1\frac{f_{\pi N\Delta}\,f_{\pi NN}}{M\,m_\pi^2}\,. \tag{A12}$$

4. The Δ excitation current of the ρ range,

$$\vec{j}_\rho^{\,a}(\Delta) \;=\; i\,\frac{q\,C_\rho^V}{9(M_\Delta - M)}\,F_1^V(q^2)\,\hat{q}\,\times\,\Big\{4\Big[1\,-\,2f(Y,Y)\Big]\,(\vec{q}_2\times(\vec{\sigma}_2\times\vec{q}_2))\;\tau_2^a$$
$$+\Big[1\,+\,4f(Y,Y)\Big]\,i(\vec{\sigma}_1\times(\vec{q}_2\times(\vec{\sigma}_2\times\vec{q}_2)))\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\}$$
$$\times\Delta_F^\rho(\vec{q}_2^{\,2})\,F_{\rho NN}^2(\vec{q}_2^{\,2})\,+\,(1\,\leftrightarrow\,2)\,, \tag{A13}$$

where

$$C_\rho^V \;=\; \frac{1+\kappa_\rho^V}{2M}\left(\frac{g_\rho\,G_1}{M}\right)^2\,. \tag{A14}$$

The current $\vec{j}^{\,a}(\pi\pi)$ of Eq. (A10) is written in such a form [25, 33] that the potential current

$$\vec{j}^{\,a}(p.c.) \;=\; \vec{j}^{\,a}(p.t.)\,+\,\vec{j}^{\,a}(\pi\pi) \tag{A15}$$

satisfies the CVC equation (1.5) for any form factor $F_{\pi NN}$, if the one-pion exchange potential also contains it. Here we use for the Gaussian form factor (1.4) the following approximation in the current (A10),

$$\frac{1}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}\Big[\Delta_F^\pi(\vec{q}_2^{\,2})F_{\pi NN}^2(\vec{q}_2^{\,2}) - \Delta_F^\pi(\vec{q}_1^{\,2})F_{\pi NN}^2(\vec{q}_1^{\,2})\Big] \;=\; \frac{1}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}\Delta_F^\pi(\vec{q}_1^{\,2})\Delta_F^\pi(\vec{q}_2^{\,2})F_{\pi NN}^2(\vec{q}_2^{\,2})$$
$$\Big\{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2}) + (\vec{q}_2^{\,2}+m_\pi^2)\left[1 - e^{-(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})/\Lambda_\pi^2}\right]\Big\} = \Delta_F^\pi(\vec{q}_1^{\,2})F_{\pi NN}^2(\vec{q}_2^{\,2})\Bigg\{\Delta_F^\pi(\vec{q}_2^{\,2}) - \frac{1}{\Lambda_\pi^2}$$
$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\left[\frac{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}{\Lambda_\pi^2}\right]^{n-1}\Bigg\} \approx \Delta_F^\pi(\vec{q}_1^{\,2})F_{\pi NN}^2(\vec{q}_2^{\,2})\Big[\Delta_F^\pi(\vec{q}_2^{\,2}) + \frac{1}{\Lambda_\pi^2}\Big(1 - \frac{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}{2\Lambda_\pi^2}\Big)\Big]\,. \tag{A16}$$

The weak axial MECs are

1. The π potential current [44],

$$\vec{j}_{5\pi}^{\,a}(p.c.) \;=\; \left(\frac{f_{\pi NN}}{m_\pi}\right)^2\frac{g_A}{2M}\,F_A(q^2)\,[\,(\vec{q}+i\vec{\sigma}_1\times\vec{P}_1)\,\tau_2^a\,+\,(\vec{P}_1+i\vec{\sigma}_1\times\vec{q})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,]$$
$$\times\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}^2(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2)\,+\,(1\,\leftrightarrow\,2)\,. \tag{A17}$$

2. The ρ-π current,

$$\begin{aligned}
\vec{j}_5^{\,a}(\rho\pi) &= -\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2 \frac{1}{4Mg_A}\,[\,1+\,m_\rho^2\,\Delta_F^\rho(\vec{q}_1^{\,2})\,]\,[\,\vec{P}_1+(1+\kappa_\rho^V)\,i\,(\vec{\sigma}_1\times\vec{q}_1)\,] \\
&\quad \times F_{\rho NN}(\vec{q}_1^{\,2})\,\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}(\vec{q}_2^{\,2})\;(\vec{\sigma}_2\cdot\vec{q}_2)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,+\,(1\,\leftrightarrow\,2) \\
&\approx \Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2 \frac{1}{4Mg_A}(1+\kappa_\rho^V)[\,1+\,m_\rho^2\,\Delta_F^\rho(\vec{q}_1^{\,2})\,]\,i\,(\vec{\sigma}_1\times\vec{q}_2) \\
&\quad \times F_{\rho NN}(\vec{q}_1^{\,2})\,\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}(\vec{q}_2^{\,2})\;(\vec{\sigma}_2\cdot\vec{q}_2)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,+\,(1\,\leftrightarrow\,2)\,. \qquad \text{(A18)}
\end{aligned}$$

Only the second part of Eq. (A18) contributes to the rate $\Lambda_{1/2}$ sensibly.

3. The Δ excitation current of the pion range,

$$\begin{aligned}
\vec{j}_{5\pi}^{\,a}(\Delta) &= \frac{g_A}{9(M_\Delta-M)}\Big(\frac{f_{\pi N\Delta}}{m_\pi}\Big)^2 F_A(q^2) \\
&\quad \times[1\,-\,\vec{q}\Delta_F^\pi(q^2)\,\vec{q}\cdot\,]\Big\{4\Big[1\,-\,\frac{1}{2}f(Z,Z)\Big]\,\vec{q}_2\,\tau_2^a \\
&\quad +\,[1\,+\,f(Z,Z)]\;i\,(\vec{\sigma}_1\times\vec{q}_2)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,\} \\
&\quad \times\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}^2(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2)\,+\,(1\,\leftrightarrow\,2)\,. \qquad \text{(A19)}
\end{aligned}$$

4. The Δ excitation current of the ρ meson range,

$$\begin{aligned}
\vec{j}_{5\rho}^{\,a}(\Delta) &= \frac{g_A\,C_\rho^A}{9(M_\Delta-M)}\,F_A(q^2) \\
&\quad \times[1\,-\,\vec{q}\Delta_F^\pi(q^2)\,\vec{q}\cdot\,]\,\{\,4\,[1\,+\,f(Y,Z)]\;\vec{q}_2\times(\vec{\sigma}_2\times\vec{q}_2)\,\tau_2^a \\
&\quad +\,[1\,-\,2f(Y,Z)]\;i\vec{\sigma}_1\times(\vec{q}_2\times(\vec{\sigma}_2\times\vec{q}_2))\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,\} \\
&\quad \times\Delta_F^\rho(\vec{q}_2^{\,2})\,F_{\rho NN}^2(\vec{q}_2^{\,2})\,+\,(1\,\leftrightarrow\,2)\,, \qquad \text{(A20)}
\end{aligned}$$

where

$$C_\rho^A\,=\,G_1\Big(\frac{g_\rho}{M}\Big)^2\,\frac{1+\kappa_\rho^V}{4}\,\frac{f_{\pi N\Delta}}{f_{\pi NN}}\,. \qquad \text{(A21)}$$

5. The potential current of the ρ range [20]

$$\begin{aligned}
\vec{j}_{5\rho}^{\,a}(p.c.) &= \Big(\frac{g_\rho}{2}\Big)^2\,\frac{(1+\kappa_\rho^V)^2}{(2M)^3}\,g_AF_A(q^2)\,\Big\{\tau_2^a\,\Big[\vec{q}\times(\vec{\sigma}_2\times\vec{q}_2)+i\vec{\sigma}_1\times(\vec{P}_1\times(\vec{\sigma}_2\times\vec{q}_2))\Big] \\
&\quad +i(\vec{\tau}_1\times\vec{\tau}_2)^a\,\Big[\vec{P}_1\times(\vec{\sigma}_2\times\vec{q}_2)+i\vec{\sigma}_1\times(\vec{q}\times(\vec{\sigma}_2\times\vec{q}_2))\Big]\Big\} \\
&\quad \times\Delta_F^\rho(\vec{q}_2^{\,2})\,F_{\rho NN}^2(\vec{q}_2^{\,2}) \\
&\quad -\Big(\frac{g_\rho}{2}\Big)^2\,\frac{(1+\kappa_\rho^V)}{(2M)^2}\,\frac{g_P(\vec{q}^{\,2})}{m_l}\,\vec{q}\,[\tau_2^a(\vec{\sigma}_1\cdot\vec{q}_2)\,+i(\vec{\tau}_1\times\vec{\tau}_2)^a\,i(\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{q}_2)] \\
&\quad \times\Delta_F^\rho(\vec{q}_2^{\,2})\,F_{\rho NN}^2(\vec{q}_2^{\,2})\,+\,(1\,\leftrightarrow\,2)\,, \qquad \text{(A22)}
\end{aligned}$$

6. The time component of the weak axial MEC

$$\begin{aligned}
\rho_5^a(\rho\pi) &= -\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2\,\frac{1}{2g_A}\,[\,1+\,m_\rho^2\,\Delta_F^\rho(\vec{q}_1^{\,2})\,]\,F_{\rho NN}(\vec{q}_1^{\,2})\,\Delta_F^\pi(\vec{q}_2^{\,2})\,F_{\pi NN}(\vec{q}_2^{\,2}) \\
&\quad \times(\vec{\sigma}_2\cdot\vec{q}_2)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,+\,(1\,\leftrightarrow\,2)\,. \qquad \text{(A23)}
\end{aligned}$$

In Eqs. (A9)-(A23), $\vec{P}_i\,=\,\vec{p}_i^{\,\prime}+\vec{p}_i$, $\vec{q}\,=\,\vec{q}_1\,+\,\vec{q}_2$, $\vec{q}_i\,=\,\vec{p}_i^{\,\prime}-\vec{p}_i$. In reaction (1.1), the isospin components of the currents and charge densities $\mathcal{O}^{\,-}\,=\,\mathcal{O}^{\,1}\,-i\,\mathcal{O}^{\,2}$ are effective.

2. The Fourier transform of the weak MECs

Here we provide the Fourier transform of the weak MECs in the same order as they are listed in the previous section. We start with presenting the form factors arising due to the form factors of the type (1.4) after the Fourier transformation of the weak MECs containing one boson propagator. They are

$$
\begin{aligned}
W_{0\,B} \;&\equiv\; \phi^0_c(x_B) \;=\; e^{(m_B/\Lambda_B)^2}\Big[e^{-x_B}\,erfc\Big(-\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big)\;-\;e^{x_B}\,erfc\Big(\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big)\Big] \\
&\quad /(2x_B)\,, \qquad\qquad \text{(A24)} \\
W_{1\,B} \;&\equiv\; -\frac{d\phi^0_c(x_B)}{dx_B} \;=\; \Big\{\phi^0_c + e^{(m_B/\Lambda_B)^2}\Big[e^{-x_B}\,erfc\Big(-\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big) \\
&\quad + e^{x_B}\,erfc\Big(\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big)\Big]/2 - \frac{\Lambda_B}{\sqrt{\pi}m_B}\,e^{-(\Lambda_B x_B/2m_B)^2}\Big\}/x_B\,, \qquad\qquad \text{(A25)} \\
W_{2\,B} \;&\equiv\; \frac{d^2\phi^0_c}{dx_B^2}-\frac{d\phi^0_c}{x_B dx_B} \;=\; \Big(1+\frac{3}{x_B^2}\Big)\phi^0_c + \frac{3}{2x_B^2}e^{(m_B/\Lambda_B)^2}\Big[e^{-x_B}\,erfc\Big(-\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big) \\
&\quad + e^{x_B}\,erfc\Big(\frac{\Lambda_B x_B}{2m_B}+\frac{m_B}{\Lambda_B}\Big)\Big] - \frac{\Lambda_B}{2\sqrt{\pi}m_B}\Big[\Big(\frac{\Lambda_B}{m_B}\Big)^2+\frac{6}{x_B^2}\Big]e^{-(\Lambda_B x_B/2m_B)^2}\,, \qquad\qquad \text{(A26)} \\
W_B \;&=\; \frac{1}{2\sqrt{\pi}}\Big(\frac{\Lambda_B}{m_B}\Big)^3 e^{-(\Lambda_B x_B/2m_B)^2}\,, \qquad\qquad \text{(A27)} \\
W_{2\,B} \;&=\; W_{0\,B} + \frac{3}{x_B}W_{1\,B} - W_B\,. \qquad\qquad \text{(A28)}
\end{aligned}
$$

In Eqs. (A24)-(A26), the function $erfc(x)$ is the complementary error function [58].
First follow the weak vector MECs:

1. The π-pair term,

$$
\vec{\tilde{j}}^{\,a}(p.t.) \;=\; -i\frac{f^2_{\pi NN}}{4\pi}\,F^V_1(q^2)\,\vec\sigma_1(\vec\sigma_2\cdot\hat r)\;i\,(\vec\tau_1\times\vec\tau_2)^a\,e^{i(\vec q\cdot\vec r_1)}\,W_{1\pi}\;+\;(1\leftrightarrow 2)\,. \qquad \text{(A29)}
$$

2. The pion-in-flight term,

$$
\begin{aligned}
\vec{\tilde{j}}^{\,a}(\pi\pi) \equiv \sum_{i=1}^4 \vec{\tilde{j}}^{\,a}_i(\pi\pi) \;&=\; \frac{1}{2\pi^{3/2}\,q}\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2\,F^V_1(q^2)\;i\,(\vec\tau_1\times\vec\tau_2)^a\,e^{i(\vec q\cdot\vec r_1)} \\
&\quad \times\sum_{i=1}^4 \vec{\mathcal{O}}_i\,f^0_{LL}(r)\;+\;(1\leftrightarrow 2)\,, \qquad \text{(A30)}
\end{aligned}
$$

where

$$
\begin{aligned}
\vec{\mathcal{O}}_1 \;&=\; -i\vec q(\vec\sigma_1\cdot\vec q)(\vec\sigma_2\cdot\nabla_{\vec r})\,, \qquad \vec{\mathcal{O}}_2 \;=\; \vec q(\vec\sigma_1\cdot\nabla_{\vec r})(\vec\sigma_2\cdot\nabla_{\vec r})\,, \\
\vec{\mathcal{O}}_3 \;&=\; (\vec\sigma_1\cdot\vec q)(\vec\sigma_2\cdot\nabla_{\vec r})\nabla_{\vec r}\,, \qquad \vec{\mathcal{O}}_4 \;=\; i(\vec\sigma_1\cdot\nabla_{\vec r})(\vec\sigma_2\cdot\nabla_{\vec r})\nabla_{\vec r}\,, \qquad \text{(A31)}
\end{aligned}
$$

and

$$
f^0_{LL}(r) \;=\; \sum_L i^L\,Y_{L0}(\hat r)\hat L F^0_{LL}\,, \qquad \text{(A32)}
$$

$$
\begin{aligned}
F^{n}_{LK} &= \int_0^{+\infty} dp\, p^{(1+n)}\, e^{-(p/\Lambda_\pi)^2}\, j_K(pr)\, \left\{ Q_L(\beta) \left[\frac{1}{p^2+m_\pi^2} \right.\right. \\
&\quad \left.\left. + \frac{1}{\Lambda_\pi^2}\left(1+\frac{p^2+m_\pi^2}{2\Lambda_\pi^2}\right)\right] - \delta_{L0}\frac{pq}{\Lambda_\pi^4} \right\} . \qquad \text{(A33)}
\end{aligned}
$$

Here $\beta = (p^2+q^2+m_\pi^2)/2pq$ and $Q_L(\beta)$ is the Legendre polynomial of the second sort [58].

Numerically, only the current $\vec{\tilde{j}}^{\,a}_3(\pi\pi)$ contributes non-negligibly to the $\Lambda_{1/2}$ for the transition $d \to 1S_0$,

$$
\begin{aligned}
\vec{\tilde{j}}^{\,a}_3(\pi\pi) &= -\frac{1}{\pi\sqrt{3}}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 F_1^V(q^2)\; i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\, e^{i(\vec{q}\cdot\vec{r}_1)} \sum_L i^L \sum_{N=L\pm1} c_N \\
&\quad \times \sum_{K=N\pm1} d_K F^2_{LK} \sum_{jgh} (-1)^g \hat{j}\hat{g}\hat{h} \begin{Bmatrix} L & 1 & N \\ K & 1 & j \end{Bmatrix} \begin{Bmatrix} K & 1 & j \\ h & g & 1 \end{Bmatrix} \\
&\quad \times [\hat{e}\otimes[[\vec{\sigma}_1\otimes\vec{\sigma}_2]^h \otimes [Y_1(\hat{q})\otimes Y_K(\hat{r})]^g]^j]^{L0} + (1 \leftrightarrow 2)\,, \qquad \text{(A34)}
\end{aligned}
$$

where $\hat{e} = \hat{e}_\pm, \hat{e}_0$ are the orthogonal unit vectors, and

$$
c_{L+1} = \sqrt{L+1}\,,\; c_{L-1} = \sqrt{L}\,,\; d_{N+1} = \sqrt{N+1}\,,\; d_{N-1} = \sqrt{N}\,. \qquad \text{(A35)}
$$

3. The Δ excitation current of the π range,

$$
\begin{aligned}
\vec{\tilde{j}}^{\,a}_\pi(\Delta) &= -i\frac{q\, C^V_\pi m_\pi^3}{36\pi(M_\Delta - M)} F_1^V(q^2) e^{i(\vec{q}\cdot\vec{r}_1)} \sum_{i=1}^{4} F^V_{i\,\pi}\hat{q}\times\vec{\mathcal{O}}^a_i(\Delta) + (1\leftrightarrow 2)\,, \qquad \text{(A36)} \\
\vec{\mathcal{O}}^a_1(\Delta) &= \vec{\sigma}_2\;\tau_2^{\,a}\,, \qquad \text{(A37)} \\
\vec{\mathcal{O}}^a_2(\Delta) &= -\hat{r}\,(\vec{\sigma}_2\cdot\hat{r})\,\tau_2^{\,a}\,, \qquad \text{(A38)} \\
\vec{\mathcal{O}}^a_3(\Delta) &= i(\vec{\sigma}_1\times\vec{\sigma}_2)\; i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, \qquad \text{(A39)} \\
\vec{\mathcal{O}}^a_4(\Delta) &= i(\hat{r}\times\vec{\sigma}_1)(\vec{\sigma}_2\cdot\hat{r})\; i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, \qquad \text{(A40)} \\
F^V_{1\,\pi} &= \frac{4}{x_\pi} W_{1\,\pi}\,[1+f(Y,Z)]\,, \quad F^V_{2\,\pi} = 4\,W_{2\,\pi}\,[1+f(Y,Z)]\,, \\
F^V_{3\,\pi} &= \frac{1}{x_\pi} W_{1\,\pi}\,[1-2f(Y,Z)]\,, \quad F^V_{4\,\pi} = W_{2\,\pi}\,[1-2f(Y,Z)]\,. \qquad \text{(A41)}
\end{aligned}
$$

4. The Δ excitation current of the ρ range,

$$
\begin{aligned}
\vec{\tilde{j}}^{\,a}_\rho(\Delta) &= -i\,\frac{q\, C^V_\rho m_\rho^3}{36\pi(M_\Delta - M)} F_1^V(q^2) e^{i(\vec{q}\cdot\vec{r}_1)} \sum_{i=1}^{4} F^V_{i\,\rho}\,\hat{q}\times\vec{\mathcal{O}}^a_i(\Delta) + (1\leftrightarrow 2)\,, \qquad \text{(A42)} \\
F^V_{1\,\rho} &= 4(W_{2\,\rho} - \frac{2}{x_\rho} W_{1\,\rho})\,[1-2f(Y,Y)]\,,\; F^V_{2\,\rho} = 4\,W_{2\,\rho}\,[1-2f(Y,Y)]\,, \\
F^V_{3\,\rho} &= (W_{2\,\rho} - \frac{2}{x_\rho} W_{1\,\rho})\,[1+4f(Y,Y)]\,,\; F^V_{4\,\rho} = W_{2\,\rho}\,[1+4f(Y,Y)]\,. \qquad \text{(A43)}
\end{aligned}
$$

Next we present the Fourier transform of the weak axial MECs:

1. The π potential current,

$$
\begin{aligned}
\vec{j}_{5\pi}^{\,a}(p.c.) &= \frac{f_{\pi NN}^2}{4\pi}\frac{m_\pi}{2M} g_A\, F_A(q^2)\, e^{i(\vec{q}\cdot\vec{r}_1)} \sum_{i=1}^{10} F_{i\pi}(p.c.)\, \vec{\mathcal{O}}_{i\pi}^a(p.c.) + (1 \leftrightarrow 2)\,, && \text{(A44)}\\
\vec{\mathcal{O}}_{1\pi}^a(p.c.) &= (\vec{q}\times\vec{\sigma}_1)\,(\vec{\sigma}_2\cdot\hat{r})\,\tau_2^{\,a}\,, \quad \vec{\mathcal{O}}_{2\pi}^a(p.c.) = 2i\,(\vec{\sigma}_2\cdot\hat{r})(\vec{\sigma}_1\times\nabla_1)\,\tau_2^{\,a}\,, && \text{(A45)}\\
\vec{\mathcal{O}}_{4\pi}^a(p.c.) &= i\vec{q}(\vec{\sigma}_2\cdot\hat{r})\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, \; \vec{\mathcal{O}}_{5\pi}^a(p.c.) = 2(\vec{\sigma}_2\cdot\hat{r})\nabla_1\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(A46)}\\
\vec{\mathcal{O}}_{3\pi}^a(p.c.) &= i\vec{q}(\vec{\sigma}_2\cdot\hat{r})\,\tau_2^{\,a}\,, \quad \vec{\mathcal{O}}_{6\pi}^a(p.c.) = (\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\cdot\hat{r})\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(A47)}\\
\vec{\mathcal{O}}_{7\pi}^a(p.c.) &= -i(\vec{\sigma}_1\times\vec{\sigma}_2)\,\tau_2^{\,a}\,, \quad \vec{\mathcal{O}}_{8\pi}^a(p.c.) = -\vec{\sigma}_2\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(A48)}\\
\vec{\mathcal{O}}_{9\pi}^a(p.c.) &= i(\vec{\sigma}_1\times\hat{r})(\vec{\sigma}_2\cdot\hat{r})\,\tau_2^{\,a}\,, \quad \vec{\mathcal{O}}_{10\pi}^a(p.c.) = \hat{r}(\vec{\sigma}_2\cdot\hat{r})\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(A49)}\\
F_{1\pi}(p.c.) &= F_{2\pi}(p.c.) = F_{3\pi}(p.c.) = F_{4\pi}(p.c.) = F_{5\pi}(p.c.) = F_{6\pi}(p.c.) &&\\
&= W_{1\pi}/m_\pi\,, && \text{(A50)}\\
F_{7\pi}(p.c.) &= F_{8\pi}(p.c.) = W_{1\pi}/x_\pi\,, && \text{(A51)}\\
F_{9\pi}(p.c.) &= F_{10\pi}(p.c.) = W_{2\pi}\,. && \text{(A52)}
\end{aligned}
$$

2. The ρ-π current,

$$
\begin{aligned}
\vec{j}_5^{\,a}(\rho\pi) &= \left(\frac{f_{\pi NN}}{m_\pi}\right)^2 \frac{1}{4Mg_A}\frac{1}{\pi q}\sqrt{\frac{3}{2\pi}}\, e^{-\frac{q^2}{2\Lambda_\rho^2}(1-a)+ia(\vec{q}\cdot\vec{r})+i(\vec{q}\cdot\vec{r}_1)}\, i(1+\kappa_\rho^V)\\
&\quad\times i\,(\vec{\tau}_1\times\vec{\tau}_2)^a[\,\Delta_1\vec{j}_5^{\,a}(\rho\pi) + \Delta_2\vec{j}_5^{\,a}(\rho\pi)\,] + (1 \leftrightarrow 2)\,. \qquad \text{(A53)}
\end{aligned}
$$

Here

$$
\begin{aligned}
\Delta_1\vec{j}_5^{\,a}(\rho\pi) &= m_\rho^2 \sum_L i^{L+1}\Big\langle \sqrt{L+1}\sum_{N=L,L+2} a_N H_{LN}^3(r) \sum_{jk}(-1)^k\hat{j}\hat{k}\\
&\quad\times\begin{Bmatrix} 1 & N & L+1\\ 1 & k & L\\ 1 & j & 1\end{Bmatrix}[\mathcal{C}(j,k,N)]^{L0} + \sqrt{L}\sum_{N=L-2,L} b_N\\
&\quad\times H_{LN}^3(r)\sum_{jk}(-1)^k\hat{j}\hat{k}\begin{Bmatrix} 1 & N & L-1\\ 1 & k & L\\ 1 & j & 1\end{Bmatrix}[\mathcal{C}(j,k,N)]^{L0}\Big\rangle\,, \qquad \text{(A54)}
\end{aligned}
$$

where

$$
\begin{aligned}
a &= \frac{\Lambda_{\pi\rho}^2}{\Lambda_\rho^2}\,, \quad \frac{1}{\Lambda_{\pi\rho}^2} \equiv \frac{1}{\Lambda_\pi^2} + \frac{1}{\Lambda_\rho^2}\,, && \text{(A55)}\\
a_L &= \sqrt{L+1}\,, \quad a_{L+2} = \sqrt{L+2}\,, \quad b_{L-2} = \sqrt{L-1}\,, \quad b_L = \sqrt{L}\,, && \text{(A56)}\\
H_{LN}^n(r) &= \int_0^{+\infty} dp\, p^n\, e^{-\frac{p^2}{2a\Lambda_\rho^2}}\, j_N(pr) F_L(p,q)\,, && \text{(A57)}\\
F_L(p,q) &= \frac{Q_L(\alpha) - Q_L(\beta)}{p^2 + a(1-a)\,q^2 + (1-a)\,m_\pi^2 + a\,m_\rho^2}\,, && \text{(A58)}\\
\alpha &= (p^2 + a^2\,q^2 + m_\pi^2)/(2apq)\,, \quad \beta = [p^2 + (1-a)^2\,q^2 + m_\rho^2] &&\\
&\quad /[2(a-1)pq]\,. && \text{(A59)}
\end{aligned}
$$

Further the symbols $[\mathcal{C}(j,k,N)]^{L0}$ are defined as

$$[\mathcal{C}(j,k,N)]^{L0} \;=\; [\hat{e}\otimes[Y_N(\hat{r})\otimes[\vec{\sigma}_1\otimes\vec{\sigma}_2]^j]^k]^{L0}\,, \tag{A60}$$

and the piece $\Delta_2\vec{\tilde{j}}_5^{\,a}(\rho\pi)$ can be obtained from the term $\Delta_1\vec{\tilde{j}}_5^{\,a}(\rho\pi)$, Eq. (A54), by the change

$$m_\rho^2 \;\to\; 1/a\,, \quad F_L(p,q) \;\to\; Q_L(\alpha)\,. \tag{A61}$$

At last, the symbols $\left\{\begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}\right\}$ in Eq. (A54) are Wigner's 9j symbols [56].

3. The Δ excitation current of the π range,

$$\begin{aligned}
\vec{\tilde{j}}_{5\pi}^{\,a}(\Delta) \;&=\; \frac{g_A F_A m_\pi^3}{36\pi(M_\Delta - M)}\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2 e^{i(\vec{q}\cdot\vec{r}_1)} \\
&\times[1-\vec{q}\Delta_F^\pi(q^2)\vec{q}\cdot]\sum_{i=1}^{4} F_{i\,\pi}^A\,\vec{\mathcal{O}}_i^a(\Delta) \;+\; (1\;\leftrightarrow\;2)\,, \qquad \text{(A62)}
\end{aligned}$$

$$\begin{aligned}
F_{1\,\pi}^A \;&=\; \frac{4}{x_\pi}W_{1\,\pi}\,[1-\frac{1}{2}f(Z,Z)]\,, \quad F_{2\,\pi}^A \;=\; 4\,W_{2\,\pi}\,[1-\frac{1}{2}f(Z,Z)]\,, \\
F_{3\,\pi}^A \;&=\; \frac{1}{x_\pi}W_{1\,\pi}\,[1+f(Z,Z)]\,, \quad F_{4\,\pi}^A \;=\; W_{2\,\pi}\,[1+f(Z,Z)]\,. \qquad \text{(A63)}
\end{aligned}$$

4. The Δ excitation current of the ρ range,

$$\begin{aligned}
\vec{\tilde{j}}_{5\rho}^{\,a}(\Delta) \;&=\; -\frac{g_A\,m_\rho^3 C_\rho^A}{36\pi(M_\Delta - M)}\,F_A(q^2)e^{i(\vec{q}\cdot\vec{r}_1)} \\
&\times[1-\vec{q}\Delta_F^\pi(q^2)\vec{q}\cdot]\sum_{i=1}^{4} F_{i\,\rho}^A\,\vec{\mathcal{O}}_i^a(\Delta) \;+\; (1\;\leftrightarrow\;2)\,, \qquad \text{(A64)}
\end{aligned}$$

$$\begin{aligned}
F_{1\,\rho}^A \;&=\; 4(W_{2\,\rho}-\frac{2}{x_\rho}W_{1\,\rho})\,[1+f(Y,Z)]\,,\; F_{2\,\rho}^A \;=\; 4\,W_{2\,\rho}\,[1+f(Y,Z)]\,, \\
F_{3\,\rho}^A \;&=\; (W_{2\,\rho}-\frac{2}{x_\rho}W_{1\,\rho})\,[1-2f(Y,Z)]\,,\; F_{4\,\rho}^A \;=\; W_{2\,\rho}\,[1-2f(Y,Z)]\,. \qquad \text{(A65)}
\end{aligned}$$

5. The potential current of the ρ range

$$\begin{aligned}
\vec{\tilde{j}}_{5\rho}^{\,a}(p.c.) \;&=\; \frac{1}{4\pi}\left(\frac{g_\rho}{2}\right)^2\left(\frac{m_\rho}{2M}\right)^3 g_A F_A(q^2)(1+\kappa_\rho^V)^2 e^{i(\vec{q}\cdot\vec{r}_1)}\sum_{i=1}^{6}F_{i\rho}(p.c.) \\
&\times\vec{\mathcal{O}}_{i\rho}^a(p.c.) \;+\; (1\;\leftrightarrow\;2)\,, \qquad \text{(A66)} \\
\vec{\mathcal{O}}_{1\rho}^a(p.c.) \;&=\; q(\vec{\sigma}_1\times\hat{r})(\vec{\sigma}_2\cdot\hat{q})\,\tau_2^a\,,\; \vec{\mathcal{O}}_{2\rho}^a(p.c.) = -2i(\vec{\sigma}_1\times\hat{r})(\vec{\sigma}_2\cdot\nabla_1)\,\tau_2^a\,, \qquad \text{(A67)} \\
\vec{\mathcal{O}}_{3\rho}^a(p.c.) \;&=\; -i(\vec{\sigma}_1\times\hat{r})(\vec{\sigma}_2\cdot\hat{r})\,\tau_2^a\,,\; \vec{\mathcal{O}}_{4\rho}^a(p.c.) = -iq\hat{r}(\vec{\sigma}_2\cdot\hat{q})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, \qquad \text{(A68)} \\
\vec{\mathcal{O}}_{5\rho}^a(p.c.) \;&=\; -2\hat{r}(\vec{\sigma}_2\cdot\nabla_1)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,,\; \vec{\mathcal{O}}_{6\rho}^a(p.c.) = -\hat{r}(\vec{\sigma}_2\cdot\hat{r})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, \qquad \text{(A69)} \\
F_{1\rho}(p.c.) \;&=\; F_{2\rho}(p.c.) = F_{4\rho}(p.c.) = F_{5\rho}(p.c.) = W_{1\rho}\,, \qquad \text{(A70)} \\
F_{3\rho}(p.c.) \;&=\; F_{6\rho}(p.c.) = m_\rho\,W_{2\rho}\,. \qquad \text{(A71)}
\end{aligned}$$

6. The time component of the weak axial MEC

$$\tilde{\rho}_5^a(\rho\pi) \;=\; -\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2 \frac{1}{2g_A}\,\frac{a}{2\pi^{3/2}} e^{-\frac{q^2}{2\Lambda_\rho^2}(1-a)+ia(\vec{q}\cdot\vec{r})+i(\vec{q}\cdot\vec{r}_1)}\,(\vec{\sigma}_2\cdot\hat{q})$$
$$\times\, i\,(\vec{\tau}_1\times\vec{\tau}_2)^a[\Delta_1\tilde{\rho}_5^a(\rho\pi) \,+\, \Delta_2\tilde{\rho}_5^a(\rho\pi)] \,+\, (1\leftrightarrow 2)\,. \qquad \text{(A72)}$$

Here

$$\Delta_1\tilde{\rho}_5^a(\rho\pi) \;=\; m_\rho^2 \sum_L i^{-L}\hat{L}\,Y_{L0}(\hat{r})\,H^1_{LL}(r)\,. \qquad \text{(A73)}$$

The function $H^n_{LN}(r)$ is defined in Eq. (A57) and the piece $\Delta_2\tilde{\rho}_5^a(\rho\pi)$ can be obtained from the term $\Delta_1{\tilde{\rho}_5}^a(\rho\pi)$, Eq. (A73), by the change (A61).

Appendix B: The EFT currents

For the one-body currents, we take the currents from Eqs. (A1)–(A4) of Appendix A, with the form factors in the quadratic radius approximation [59, 60],

$$F_1^V(q^2) \;\approx\; 1-\frac{1}{6}r_V^2\,q^2\,,\quad r_V^2 \;=\; 0.59\ \text{fm}^2\,, \qquad \text{(B1)}$$

$$F_A(q^2) \;\approx\; 1-\frac{1}{6}r_A^2\,q^2\,,\quad r_A^2 \;=\; (0.403\pm 0.030)\ \text{fm}^2\,, \qquad \text{(B2)}$$

However, this approximation changes the results only a little bit in comparison with the dipole form factors used in the TAA calculations.

1. The weak exchange currents

For the weak vector currents, we take the currents from Eqs. (A9)–(A13) of Appendix A. So we add to the π-pair and pion-in-flight terms considered in [13] the Δ excitation currents of the π and ρ ranges. Inspecting Table 2 of Ref. [46] one can in addition expect a non-negligible contribution also from the $\rho\omega\pi$ and two-pion exchange currents.
As to the weak axial MEC operator, we adopt here the main part of this current used in [13] and add to it the π potential current of Eq. (A17) demanded by the PCAC constraint (1.6). In our notation, the currents (19)–(21) of Ref. [13] are:

$$\hat{A}^0_{2B} \;=\; -\frac{g_A F_A}{4f_\pi^2}\Delta_F^\pi(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2)\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a \,+\,(1\leftrightarrow 2)\,, \qquad \text{(B3)}$$

$$\begin{aligned}\hat{\vec{A}}_{2B} \;=\; \frac{g_A F_A}{2Mf_\pi^2}\Big\{&\Big[\frac{1}{4}\vec{P}_1\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a + 2\hat{c}_3\,\vec{q}_2\,{\tau_2}^a - (\hat{c}_4+\frac{1}{4})i(\vec{\sigma}_1\times\vec{q}_2)\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a \\ &+\frac{1+c_6}{4}\,i(\vec{\sigma}_1\times\vec{q})\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big]\,\Delta_F^\pi(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2) \\ &+\hat{d}_1(\vec{\sigma}_1\,{\tau_1}^a+\vec{\sigma}_2\,{\tau_2}^a)\,-\,\hat{d}_2\,i(\vec{\sigma}_1\times\vec{\sigma}_2)\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\Big\}\,+\,(1\leftrightarrow 2)\,,\end{aligned} \qquad \text{(B4)}$$

$$\hat{P}(\vec{q}_1,\vec{q}_2) \;=\; \frac{g_A F_A m_\pi^2}{2Mf_\pi^2}\,4\hat{c}_1\;{\tau_2}^a\,\Delta_F^\pi(\vec{q}_2^{\,2})\,(\vec{\sigma}_2\cdot\vec{q}_2)\,+\,(1\leftrightarrow 2)\,. \qquad \text{(B5)}$$

In Eq. (B3), we keep only the leading order of the time component considered in Eq. (18) [13]. because the contribution of this part of $\hat{A}^0_{2B}$ is small and the correction to it is suppressed by the factor $\approx 1/\mathrm{M}^2$. Let us note that the time component (B3) is the soft pion approximation to its hard pion form (A23).

2. The Fourier transform of the EFT weak axial MECs

Multiplying the currents (B3), (B4) and (B5) by the form factor squared of the type (1.4) we obtain

$$
\begin{aligned}
\tilde{\hat{A}}^0_{2B} &= -i\frac{1}{4\pi}\frac{g_A F_A m_\pi^2}{4f_\pi^2}(\vec{\sigma}_2\cdot\hat{r})e^{i(\vec{q}\cdot\vec{r}_1)}\,W_{1\pi}\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\;+\;(1\leftrightarrow 2)\,, && \text{(B6)}\\
\tilde{\hat{\vec{A}}}_{2B} &= \frac{1}{4\pi}\frac{g_A F_A m_\pi^3}{2Mf_\pi^2}e^{i(\vec{q}\cdot\vec{r}_1)}\sum_{i=1}^{10}F_i(2B)\,\vec{\mathcal{O}}^a_i(2B)\;+(1\leftrightarrow 2)\,, && \text{(B7)}\\
\vec{\mathcal{O}}^a_1(2B) &= i\vec{q}\,(\vec{\sigma}_2\cdot\hat{r})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,,\quad \vec{\mathcal{O}}^a_2(2B)\;=\;2\,(\vec{\sigma}_2\cdot\hat{r})\,\nabla_1\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(B8)}\\
\vec{\mathcal{O}}^a_3(2B) &= -\vec{\sigma}_2\;i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,,\quad \vec{\mathcal{O}}^a_4(2B)\;=\;\hat{r}(\vec{\sigma}_2\cdot\hat{r})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(B9)}\\
\vec{\mathcal{O}}^a_5(2B) &= 2\vec{\sigma}_2\;{\tau_2}^a\,,\quad \vec{\mathcal{O}}^a_6(2B)=-2\hat{r}(\vec{\sigma}_2\cdot\hat{r})\,{\tau_2}^a\,, && \text{(B10)}\\
\vec{\mathcal{O}}^a_7(2B) &= -i(\vec{\sigma}_1\times\vec{\sigma}_2)\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,,\quad \vec{\mathcal{O}}^a_8(2B)\;=\;i(\vec{\sigma}_1\times\hat{r})(\vec{\sigma}_2\cdot\hat{r})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,, && \text{(B11)}\\
\vec{\mathcal{O}}^a_9(2B) &= -(\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\cdot\hat{r})\,i\,(\vec{\tau}_1\times\vec{\tau}_2)^a\,,\quad \vec{\mathcal{O}}^a_{10}(2B)\;=\;\vec{\sigma}_1\,{\tau_1}^a\,, && \text{(B12)}\\
F_1(2B) &= F_2(2B)\;=\;W_{1\pi}/4m_\pi\,,\quad F_3(2B)\;=\;W_{1\pi}/4x_\pi\,,\quad F_4(2B)\;=\;W_{2\pi}/4\,,\\
F_5(2B) &= \hat{c}_3\,W_{1\pi}/x_\pi\;+\;\hat{d}_1 W_\pi/2\,,\quad F_6(2B)\;=\;\hat{c}_3\,W_{2\pi}\,,\\
F_7(2B) &= (\hat{c}_4+\frac{1}{4})\,W_{1\pi}/x_\pi+\hat{d}_2 W_\pi\,,\quad F_8(2B)\;=\;(\hat{c}_4+\frac{1}{4})\,W_{2\pi}\,,\\
F_9(2B) &= \frac{1+c_6}{4}\,W_{1\pi}/m_\pi\,,\quad F_{10}(2B)\;=\;\hat{d}_1\,W_\pi\,. && \text{(B13)}\\
\tilde{\hat{P}}(\vec{q}_1,\vec{q}_2) &= i\frac{1}{4\pi}\frac{2g_A F_A m_\pi^2}{Mf_\pi^2}\,\hat{c}_1\,(\vec{\sigma}_2\cdot\hat{r})e^{i(\vec{q}\cdot\vec{r}_1)}\,W_{1\pi}\;{\tau_2}^a\;+\;(1\leftrightarrow 2)\,. && \text{(B14)}
\end{aligned}
$$

Appendix C: The multipoles of the currents

We first present the contribution to the multipoles from the IA currents. In order to make the equations more transparent, we do not write the argument $qr/2$ in the Bessel functions $j_i(qr/2)$, unless the argument differs, which is the case of the ρ-π current. The factor $\sqrt{2}$ arising from the isovector matrix elements is not kept in the reduced matrix elements of the current, but is included in the overall constants in front of the integrals in Eqs. (3.2) and (3.7). We also take into account the factor $1/\kappa_0$, entering the reduced matrix elements according to the definition given in Eq. (4.20) of Ref. [20], by keeping $1/\kappa_0^2$ in the integration volume in the same Eqs. (3.2) and (3.7).

1. The multipoles of the IA currents

a. *J=0 multipoles*

$$
<^3 P_1||\hat{L}_{50}||d>\;=\;i\sqrt{2}[-g_A F_A(1-\frac{\vec{q}^{\,2}}{8M^2})+\frac{g_P\,\vec{q}^{\,2}}{2Mm_\mu}]\int_0^{+\infty}dr\,u^1_{11,2}(\kappa,r)\,j_1\,(u_0(r)
$$

$$
\begin{aligned}
&+u_2(r)/\sqrt{2}\Big) + i\sqrt{2}\frac{g_A F_A}{M^2}\left\langle \frac{q}{6}\int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\ \{(j_0+j_2)\,(u_0'(r)\right. \\
&-u_0(r)/r) + [(j_0+j_2)u_2'(r) + (2j_0 - 5j_2/2)u_2(r)/r]\sqrt{2}\Big\} \\
&+\int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_1\Big[-u_0'(r)/r + u_0(r)/r^2 + (u_2'(r)/2 \\
&\left. +u_2(r)/r)\,/\sqrt{2}r\Big]\right\rangle . \qquad \text{(C1)}
\end{aligned}
$$

$$
\begin{aligned}
<^3 P_1||\hat{M}_{50}||d> \;=\;& i\frac{\sqrt{2}q}{2M}[g_A F_A - \frac{g_P\, q_0}{m_\mu}] \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_1(u_0(r) + u_2(r)/\sqrt{2}) \\
&-i\sqrt{2}\,\frac{g_A F_A}{M}\,\int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_0\,[u'(r) - u_0(r)/r + (u_2'(r) \\
&+2u_2(r)/r)\,/\sqrt{2}\Big] . \qquad \text{(C2)}
\end{aligned}
$$

b. J=1 multipoles

$$
<^1 S_0||\hat{T}_1^{mag}||d> = -i\frac{q\, G_M^V}{\sqrt{2}\, M}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[j_0 u_0(r) - j_2 u_2(r)/\sqrt{2}]\,, \qquad \text{(C3)}
$$

$$
\begin{aligned}
<^1 S_0||i\hat{T}^{el}_{51}||d> \;=\;& -i\sqrt{2}g_A F_A(1-\frac{\vec{q}^{\,2}}{8M^2})\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\ [j_0 u_0(r) \\
&-j_2 u_2(r)/\sqrt{2}\Big] + i\frac{g_A F_A}{M^2}\left\{\frac{q}{\sqrt{2}}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\right. \\
&j_1[u_0'(r) - u_0(r)/r] + \frac{q}{20}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[(7j_1 \\
&-3j_3)\,u_2'(r) + (14j_1 + 9j_3)u_2(r)/r] \\
&+\frac{1}{3\sqrt{2}}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[(-2j_0 + j_2)u_0''(r) + 3j_2\,(-u_0'(r)/r \\
&+u_0(r)/r^2\Big)\Big] - \frac{1}{3}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[(j_0 - j_2/2)u_2''(r) \\
&\left.+3j_0 u_2'(r)/r + 3j_2 u_2(r)/r^2\Big]\right\} , \qquad \text{(C4)}
\end{aligned}
$$

$$
\begin{aligned}
<^1 S_0||\hat{L}_{\,51}||d> \;=\;& -i[g_A F_A(1-\frac{\vec{q}^{\,2}}{8M^2}) - \frac{g_P\,\vec{q}^{\,2}}{2M\, m_\mu}]\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\ [j_0 u_0(r) \\
&+\sqrt{2}j_2 u_2(r)\Big] + i\frac{g_A F_A}{2M^2}\left\{\frac{3q}{5\sqrt{2}}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\right. \\
&[(j_1 + j_3)u_2'(r) + (2j_1 - 3j_3)u_2(r)/r] \\
&-\frac{2}{3}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[(j_0 + j_2)u_0''(r) + 3j_2\,(-u_0'(r)/r \\
&+u_0(r)/r^2\Big)\Big] - \frac{\sqrt{2}}{3}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,[(j_0 + j_2)u_2''(r) \\
&\left.+3j_0 u_2'(r)/r - 6j_2 u_2(r)/r^2\Big]\right\} , \qquad \text{(C5)}
\end{aligned}
$$

$$
\begin{aligned}
<^1 S_0||\hat{M}_{51}||d> \;&=\; i\frac{q}{2M}(g_A F_A - \frac{g_P\, q_0}{m_\mu}) \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, \left[j_0 u_0(r) + \sqrt{2} j_2 u_2(r)\right] \\
&\quad + i\frac{g_A F_A}{M} \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, j_1\, \left[u_0'(r) - u_0(r)/r - \sqrt{2}\,(u_2'(r)\right. \\
&\quad \left. + 2u_2(r)/r)\right] , \qquad \text{(C6)}
\end{aligned}
$$

$$
\begin{aligned}
<^3 P_0||\hat{T}^{mag}_{51}||d> \;&=\; \sqrt{2} g_A F_A I^1_1 , \quad <^3 P_0||i\hat{T}^{el}_1||d> = \frac{\sqrt{2}}{M}[\frac{q\, G^V_M}{2} I^1_1 - \frac{F^V_1}{3} K^1_1] , \qquad \text{(C7)} \\
<^3 P_0||\hat{M}_1||d> \;&=\; -F^V_1 J^1_1 , \quad <^3 P_0||\hat{L}_1||d> = \frac{F^V_1}{M}[\frac{q}{2} J^1_1 - \frac{1}{3} K^1_2] , \qquad \text{(C8)} \\
<^3 P_1||\hat{T}^{mag}_{51}||d> \;&=\; -\sqrt{\frac{3}{2}} g_A F_A I^1_2 , \quad <^3 P_1||i\hat{T}^{el}_1||d> = \frac{1}{M}\left[-\sqrt{\frac{3}{2}} \frac{q\, G^V_M}{2} I^1_2 \right. \\
&\quad \left. + \sqrt{\frac{2}{3}} F^V_1 K^1_3 \right] , \quad <^3 P_1||\hat{M}_1||d> = \sqrt{3} F^V_1 J^1_2 , \qquad \text{(C9)}
\end{aligned}
$$

$$
\begin{aligned}
<^3 P_1||\hat{L}_1||d> \;&=\; \sqrt{3}\frac{F^V_1}{M}\left[-\frac{q}{2} J^1_2 + \frac{1}{3} K^1_4\right] , <^3 P_2||\hat{T}^{mag}_{51}||d> = -\sqrt{\frac{5}{2}} g_A F_A I^1_{3\lambda} , \qquad \text{(C10)} \\
<^3 P_2||i\hat{T}^{el}_1||d> \;&=\; -\sqrt{\frac{5}{2}}\frac{1}{M}\left[\frac{q\, G^V_M}{2} I^1_{3\lambda} + \frac{2}{3} F^V_1 K^1_{5\lambda}\right] , \qquad \text{(C11)} \\
<^3 P_2||\hat{M}_1||d> \;&=\; -\sqrt{5} F^V_1 J^1_{3\lambda} , \quad <^3 P_2||\hat{L}_1||d> = \frac{\sqrt{5}}{M} F^V_1 \left[\frac{q}{2} J^1_{3\lambda} - \frac{1}{3} K^1_{6\lambda}\right] , \qquad \text{(C12)} \\
<^3 F_2||\hat{T}^{mag}_{51}||d> \;&=\; 3\sqrt{\frac{3}{10}} g_A F_A I^1_{4\lambda} , \quad <^3 F_2||i\hat{T}^{el}_1||d> = \sqrt{\frac{3}{10}}\frac{1}{M}\left[\frac{3q}{2} G^V_M I^1_{4\lambda}\right. \\
&\quad \left. - 2F^V_1 K^1_{7\lambda}\right] , \quad <^3 F_2||\hat{M}_1||d> = -3\sqrt{\frac{3}{5}} F^V_1 J^1_{4\lambda} , \qquad \text{(C13)} \\
<^3 F_2||\hat{L}_1||d> \;&=\; \sqrt{\frac{3}{5}}\frac{F^V_1}{M}\left[\frac{3q}{2} J^1_{4\lambda} - K^1_{8\lambda}\right] , \qquad \text{(C14)} \\
<^1 D_2||i\hat{T}^{el}_{51}||d> \;&=\; i g_A F_A I^1_5 , \quad <^1 D_2||\hat{T}^{mag}_1||d> = i\frac{q\, G^V_M}{2M} I^1_5 , \qquad \text{(C15)} \\
<^1 D_2||\hat{M}_{51}||d> \;&=\; i\frac{q}{\sqrt{2}M}(g_A F_A - \frac{q_0 g_P}{m_\mu}) I^1_6 - i\sqrt{2}\frac{g_A F_A}{M} I^1_7 , \qquad \text{(C16)} \\
<^1 D_2||\hat{L}_{51}||d> \;&=\; -i\sqrt{2}(g_A F_A - \frac{\vec{q}^{\,2} g_P}{2M m_\mu}) I^1_6 , \qquad \text{(C17)}
\end{aligned}
$$

$$
\begin{aligned}
I^1_1 \;&=\; \int_0^{+\infty} dr\, u^0_{11,2}(\kappa,r)\, j_1\, [u_0(r) + u_2(r)/\sqrt{2}] , \qquad \text{(C18)} \\
I^1_2 \;&=\; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_1\, [u_0(r) - \sqrt{2} u_2(r)] , \qquad \text{(C19)} \\
I^1_{3\lambda} \;&=\; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, j_1\, [u_0(r) - \frac{2\sqrt{2}}{5} u_2(r)] , \qquad \text{(C20)}
\end{aligned}
$$

$$I^1_{4\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)\, j_1\, u_2(r)\,, \tag{C21}$$

$$I^1_5 \;=\; \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r)\, [j_2 u_0(r) - (2j_0 + j_2) u_2(r)/\sqrt{2}]\,, \tag{C22}$$

$$I^1_6 \;=\; \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r)\, [j_2 u_0(r) + (j_0 - j_2) u_2(r)/\sqrt{2}]\,, \tag{C23}$$

$$I^1_7 \;=\; \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r)\, j_1\, \left[D^1_+ u_0(r) - \sqrt{2} D^1_- u_2(r)\right]\,, \tag{C24}$$

$$J^1_1 \;=\; \int_0^{+\infty} dr\, u^0_{11,2}(\kappa,r)\, j_1\, [u_0(r) - \sqrt{2} u_2(r)]\,, \tag{C25}$$

$$J^1_2 \;=\; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_1\, [u_0(r) + u_2(r)/\sqrt{2}]\,, \tag{C26}$$

$$J^1_{3\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, j_1\, [u_0(r) - \frac{1}{5\sqrt{2}} u_2(r)]\,, \tag{C27}$$

$$J^1_{4\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)\, j_1\, u_2(r)\,, \tag{C28}$$

$$K^1_1 \;=\; \int_0^{+\infty} dr\, u^0_{11,2}(\kappa,r) \left\{ (j_0 + j_2) D^1_+ u_0(r) - \frac{1}{5\sqrt{2}} \left[(10 j_0 + j_2) D^1_- + 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C29}$$

$$K^1_2 \;=\; \int_0^{+\infty} dr\, u^0_{11,2}(\kappa,r) \left\{ (j_0 - 2j_2) D^1_+ u_0(r) - \frac{\sqrt{2}}{5} \left[(5 j_0 - j_2) D^1_- - 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C30}$$

$$K^1_3 \;=\; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r) \left\{ (j_0 + j_2) D^1_+ u_0(r) + \frac{1}{10\sqrt{2}} \left[(10 j_0 + j_2) D^1_- + 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C31}$$

$$K^1_4 \;=\; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r) \left\{ (j_0 - 2j_2) D^1_+ u_0(r) + \frac{1}{5\sqrt{2}} \left[(5 j_0 - j_2) D^1_- - 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C32}$$

$$K^1_{5\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r) \left\{ (j_0 + j_2) D^1_+ u_0(r) - \frac{1}{50\sqrt{2}} \left[(10 j_0 + j_2) D^1_- + 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C33}$$

$$K^1_{6\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r) \left\{ (j_0 - 2j_2) D^1_+ u_0(r) - \frac{1}{25\sqrt{2}} \left[(5 j_0 - j_2) D^1_- - 9 j_2 D^3_+ \right] u_2(r) \right\}\,, \tag{C34}$$

$$K^1_{7\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r) \left[j_0 D^3_+ + j_2 \frac{d}{dr} \right] u_2(r)\,, \tag{C35}$$

$$K^1_{8\lambda} \;=\; \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r) \left[j_0 D^3_+ - 2 j_2 \frac{d}{dr} \right] u_2(r)\,, \tag{C36}$$

$$D^1_+ \; = \; \frac{d}{dr} - \frac{1}{r}\,, \quad D^1_- = \frac{d}{dr} + \frac{2}{r}\,, \quad D^3_+ = \frac{d}{dr} - \frac{3}{r}\,. \tag{C37}$$

c. J=2 multipoles

$$<^1 D_2||i\hat{T}^{el}_2||d> \; = \; -\sqrt{5}\frac{q\,G^V_M}{2M}I^2_1\,, \quad <^1 D_2||\hat{T}^{mag}_{52}||d>= -\sqrt{5}g_A F_A I^2_1\,, \tag{C38}$$

$$<^3 P_1||i\hat{T}^{el}_{52}||d> \; = \; i\sqrt{\frac{3}{2}}g_A F_A I^2_2\,, \quad <^3 P_1||\hat{T}^{mag}_2||d>= i\frac{3^{3/2}}{2M}[\frac{q\,G^V_M}{3\sqrt{2}}I^2_2 + F^V_1 I^2_{11}], \tag{C39}$$

$$\begin{aligned}<^3 P_1||\hat{L}_{52}||d> \; &= \; i(g_A F_A - \frac{\vec{q}^{\,2} g_P}{2M m_\mu})I^2_5\,, <^3 P_1||\hat{M}_{52}||d>= -i\frac{q}{2M}(g_A F_A - \frac{q_0 g_P}{m_\mu})I^2_5 \\ &-i\frac{g_A F_A}{M}I^2_6\,, \quad <^3 P_2||i\hat{T}^{el}_{52}||d>= -i\frac{3}{\sqrt{2}}g_A F_A I^2_{3\lambda}\,,\end{aligned} \tag{C40}$$

$$\begin{aligned}<^3 P_2||\hat{L}_{52}||d> \; &= \; -i\sqrt{3}\left(g_A F_A - \frac{\vec{q}^{\,2} g_P}{2M m_\mu}\right) I^2_{7\lambda}\,, <^3 P_2||\hat{T}^{mag}_2||d>= i\frac{3}{2M} \\ &\left[-\frac{q}{\sqrt{2}}G^V_M I^2_{3\lambda} + F^V_1 I^2_{12\lambda}\right]\,,\end{aligned} \tag{C41}$$

$$<^3 P_2||\hat{M}_{52}||d> \; = \; i\sqrt{3}\frac{q}{2M}(g_A F_A - \frac{q_0 g_P}{m_\mu})I^2_{7\lambda} + i\sqrt{3}\frac{g_A F_A}{M}I^2_{9\lambda}\,, \tag{C42}$$

$$\begin{aligned}<^3 F_2||i\hat{T}^{el}_{52}||d> \; &= \; -i\frac{2}{\sqrt{3}}g_A F_A I^2_{4\lambda}\,, <^3 F_2||\hat{T}^{mag}_2||d>= -i\frac{1}{\sqrt{3}M}\left[q\,G^V_M I^2_{4\lambda}\right. \\ &\left.+6\sqrt{2}F^V_1 I^2_{13\lambda}\right]\,,\end{aligned} \tag{C43}$$

$$\begin{aligned}<^3 F_2||\hat{L}_{52}||d> \; &= \; i\sqrt{2}(g_A F_A - \frac{\vec{q}^{\,2} g_P}{2M m_\mu})I^2_{8\lambda}\,, <^3 F_2||\hat{M}_{52}||d>= -i\frac{q}{\sqrt{2}M}\left[g_A F_A\right. \\ &\left.-\frac{q_0 g_P}{m_\mu}\right] I^2_{8\lambda} + i\sqrt{2}\frac{g_A F_A}{M}I^2_{10\lambda}\,,\end{aligned} \tag{C44}$$

$$I^2_1 \; = \; \int_0^{+\infty} dr\, u^2_{20,1}(\kappa, r) j_2[u_0(r) + u_2(r)/\sqrt{2}]\,, \tag{C45}$$

$$I^2_2 \; = \; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa, r)[j_1 u_0(r) + \sqrt{2}(-2j_1 + 3j_3)u_2(r)/5]\,, \tag{C46}$$

$$I^2_{3\lambda} \; = \; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa, r)[j_1 u_0(r) + \sqrt{2}(j_1 + j_3)u_2(r)/5]\,, \tag{C47}$$

$$I^2_{4,\lambda} \; = \; \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa, r)[j_3 u_0(r) + (-9j_1 + 16j_3)u_2(r)/10\sqrt{2}]\,, \tag{C48}$$

$$I^2_5 \; = \; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa, r)[j_1 u_0(r) - (4j_1 + 9j_3)u_2(r)/5\sqrt{2}]\,, \tag{C49}$$

$$I^2_6 \; = \; \int_0^{+\infty} dr\, u^1_{11,2}(\kappa, r)\, j_2\, [D^1_+ u_0(r) + D^1_- u_2(r)/\sqrt{2}]\,, \tag{C50}$$

$$I^2_{7\lambda} \; = \; \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa, r)[j_1 u_0(r) + \sqrt{2}(j_1 - \frac{3}{2}j_3)u_2(r)/5]\,, \tag{C51}$$

$$I^2_{8\lambda} = \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)[j_3 u_0(r) + (3j_1+8j_3)u_2(r)/5\sqrt{2}]\,, \tag{C52}$$

$$I^2_{9\lambda} = \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, j_2\, [D^1_+ u_0(r) + D^1_- u_2(r)/\sqrt{2}]\,, \tag{C53}$$

$$I^2_{10\lambda} = \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)\, j_2\, [D^1_+ u_0(r) + D^1_- u_2(r)/\sqrt{2}]\,, \tag{C54}$$

$$I^2_{11} = \int_0^{+\infty} dr\, u^1_{11,2}(\kappa,r)\, j_2\, u_2(r)/r\,, \tag{C55}$$

$$I^2_{12\lambda} = \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, j_2\, u_2(r)/r\,, \tag{C56}$$

$$I^2_{13\lambda} = \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)\, j_2\, u_2(r)/r\,. \tag{C57}$$

d. J=3 multipoles

$$\begin{aligned} <^3 P_2||\hat{T}^{mag}_{53}||d> &= -\frac{6}{\sqrt{5}} g_A F_A I^3_{1\lambda}\,, \quad <^3 P_2||i\hat{T}^{el}_3||d> = -\frac{6}{\sqrt{5}M}\left[\frac{q\, G^V_M}{2} I^3_{1\lambda}\right. \\ &\left. + \frac{F^V_1}{35} I^3_{6\lambda}\right], <^3 P_2||\hat{L}_3||d> = -\frac{3\sqrt{3}}{35\sqrt{5}M} F^V_1 I^3_{7\lambda}\,, \end{aligned} \tag{C58}$$

$$\begin{aligned} <^3 F_2||\hat{T}^{mag}_{53}||d> &= 2\sqrt{\frac{5}{3}} g_A F_A I^3_{2\lambda}\,, <^3 F_2||i\hat{T}^{el}_3||d> = \frac{\sqrt{5}}{M}\left[\frac{q}{\sqrt{3}}\, G^V_M I^3_{2\lambda}\right. \\ &\left. -\frac{2}{7}\, F^V_1 I^3_{8\lambda}\right]\,, \quad <^3 F_2||\hat{L}_3||d> = -\frac{3\sqrt{5}}{7M} F^V_1 I^3_{9\lambda}\,, \end{aligned} \tag{C59}$$

$$<^1 D_2||\hat{T}^{mag}_3||d> = -2i\frac{q\, G^V_M}{2M} I^3_3\,, \quad <^1 D_2||i\hat{T}^{el}_{53}||d> = -2i g_A F_A I^3_3\,, \tag{C60}$$

$$\begin{aligned} <^1 D_2||\hat{L}_{53}||d> &= -i\sqrt{3}[g_A F_A - \frac{\vec{q}^{\,2} g_P}{2M m_\mu}] I^3_4\,, \quad <^1 D_2||\hat{M}_{53}||d> = i\frac{\sqrt{3}q}{2M}\left[g_A F_A\right. \\ &\left. -\frac{q_0 g_P}{m_\mu}\right] I^3_4 + \sqrt{3}\frac{g_A F_A}{M} I^3_5\,, \end{aligned} \tag{C61}$$

$$I^3_{1\lambda} = \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, j_3 u_2(r)\,, \tag{C62}$$

$$I^3_{2\lambda} = \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r)\, j_3[u_0(r) - u_2(r)/5\sqrt{2}]\,, \tag{C63}$$

$$I^3_3 = \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r)[j_2 u_0(r) - \sqrt{2}(j_2 + \frac{9}{2} j_4) u_2(r)/7]\,, \tag{C64}$$

$$I^3_4 = \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r)[j_2 u_0(r) - \sqrt{2}(j_2 - 6 j_4) u_2(r)/7]\,, \tag{C65}$$

$$I^3_5 = \int_0^{+\infty} dr\, u^2_{20,1}(\kappa,r) j_3 [D^1_+ u_0(r) - \sqrt{2} D^1_- u_2(r)]\,, \tag{C66}$$

$$I^3_{6\lambda} = \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, [14 j_2 D^1_- + (j_2 + 15 j_4) D^3_+] u_2(r)\,, \tag{C67}$$

$$I^3_{7\lambda} = \int_0^{+\infty} dr\, u^2_{11,\lambda}(\kappa,r)\, [14 j_2 D^1_- + (j_2 - 20 j_4) D^3_+] u_2(r)\,, \tag{C68}$$

$$\begin{aligned} I^3_{8\lambda} &= \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r) \left\{ (j_2 + j_4) D^1_+ u_0(r) - \sqrt{2}\frac{4}{25}\left[(j_2 + \frac{15}{8} j_4) D^3_+ \right.\right. \\ &\quad \left.\left. + \frac{1}{2}(3 j_2 + \frac{5}{4} j_4) D^1_- \right] u_2(r) \right\}\,, \end{aligned} \tag{C69}$$

$$\begin{aligned} I^3_{9\lambda} &= \int_0^{+\infty} dr\, u^2_{31,\lambda}(\kappa,r) \left\{ (j_2 - \frac{4}{3} j_4) D^1_+ u_0(r) - \sqrt{2}\frac{4}{25}\left[(j_2 - \frac{5}{2} j_4) D^3_+ \right.\right. \\ &\quad \left.\left. + \frac{1}{2}(3 j_2 - \frac{5}{3} j_4) D^1_- \right] u_2(r) \right\}\,, \end{aligned} \tag{C70}$$

We now present the multipoles J=1 of the TAA MECs given in Appendices A 1 and A 2.

2. Multipoles J=1 of the TAA MECs

First follow the multipoles of the weak vector MECs.

1. The π-pair term $\vec{j}^{\,a}(p.t.)$, Eq. (A29):

$$<^1 S_0||\hat{T}^{mag}_1||d> = -i\frac{\sqrt{2}}{\pi}\, f^2_{\pi NN}\, F^V_1 \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, j_1\, W_{1\pi}[u_0(r) + u_2(r)/\sqrt{2}]\,. \tag{C71}$$

2. The pion-in-flight term $\vec{j}^{\,a}(\pi\pi)$, Eq. (A30):

$$\begin{aligned} <^1 S_0||\hat{T}^{mag}_1||d> &= i\frac{\sqrt{2}}{3\pi^2}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 F^V_1 \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r) \left\{ \left[j_0(F^2_{00} - F^2_{20}) \right.\right. \\ &\quad \left. + j_2(F^2_{22} - F^2_{02}) \right] u_0(r) - \left[j_2(F^2_{00} - F^2_{20}) \right. \\ &\quad \left.\left. + (j_0 + 2 j_2)(F^2_{02} - F^2_{22}) \right] u_2(r)/\sqrt{2} \right\} \end{aligned} \tag{C72}$$

3. The Δ excitation current of the π range $\vec{j}^{\,a}_\pi(\Delta)$, Eq. (A36):

$$\begin{aligned} <^1 S_0||\hat{T}^{mag}_1||d> &= i\frac{\sqrt{2} q C^V_\pi m^3_\pi F^V_1}{9\pi(M_\Delta - M)} \left\{ \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\pi} \right. \\ &\quad \times \left[j_2 u_0(r) - (j_0 + j_2/2)\sqrt{2}\, u_2(r) \right] \\ &\quad - f(Y,Z) \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, (W_{2\pi} - 3W_{1\pi}/x_\pi) \\ &\quad \left. \times [-2 j_0 u_0(r) + \sqrt{2} j_2 u_2(r)] \right\}\,, \end{aligned} \tag{C73}$$

4. The Δ excitation current of the ρ range $\vec{j}^{\,a}_\rho(\Delta)$, Eq. (A42):

$$\begin{aligned} <^1 S_0||\hat{T}^{mag}_1||d> &= i\frac{\sqrt{2} q C^V_\rho m^3_\rho F^V_1}{9\pi(M_\Delta - M)} \left\{ \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\rho} \right. \\ &\quad \times \left[j_2 u_0(r) - (j_0 + j_2/2)\sqrt{2}\, u_2(r) \right] \end{aligned}$$

$$
\begin{aligned}
&+8f(Y,Y)\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,(W_{2\rho}-3W_{1\rho}/x_\rho) \\
&\times[j_0 u_0(r) - j_2 u_2(r)/\sqrt{2}]\Big\}\;, \qquad \text{(C74)}
\end{aligned}
$$

Now we write down the multipoles of the weak axial MECs,

1. The π potential current $\vec{j}^{\,a}_{5\pi}(p.c.)$, Eq. (A44):

$$
\begin{aligned}
<^1 S_0||i\hat{T}^{el}_{51}||d> \;=\;& i\frac{f^2_{\pi NN}}{2\sqrt{2}\pi}\frac{m_\pi}{M}g_A\,F_A\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\;\langle[(qj_1 \\
&-2j_2D^1_+\Big)\,W_{1\pi}/m_\pi + j_2W_{2\pi}]u_0(r) + \{[qj_1/2 \\
&+2j_0D^1_- + j_2(2D^1_- + 3D^3_+)/5\Big]\,W_{1\pi}/m_\pi \\
&-(2j_0+j_2)W_{2\pi}/2\}\,\sqrt{2}\,u_2(r)\Big\rangle\;, \qquad \text{(C75)}
\end{aligned}
$$

$$
\begin{aligned}
<^1 S_0||\hat{L}_{51}||d> \;=\;& i\frac{f^2_{\pi NN}}{4\pi}\frac{m_\pi}{M}g_A\,F_A\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\;\langle[(qj_1 \\
&+4j_2D^1_+\Big)\,W_{1\pi}/m_\pi - 2j_2W_{2\pi}]u_0(r) + \{[-qj_1 \\
&+2j_0D^1_- - 2j_2(2D^1_- + 3D^3_+)/5\Big]\,W_{1\pi}/m_\pi \\
&+(j_2-j_0)W_{2\pi}\}\,\sqrt{2}\,u_2(r)\Big\rangle\;, \qquad \text{(C76)}
\end{aligned}
$$

2. The ρ-π current $\vec{j}^{\,a}_5(\rho\pi)$, Eq. (A53):

$$
<^1 S_0||i\hat{T}^{el}_{51}||d> \;=\; i\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2\frac{(1+\kappa^V_\rho)m^2_\rho}{6\sqrt{2}\pi^2Mg_Aq}\,e^{-q^2(1-a)/2\Lambda^2_\rho}[2\bar{I}^0_1+\bar{I}^2_1]\,, \qquad \text{(C77)}
$$

$$
<^1 S_0||\hat{L}_{51}||d> \;=\; i\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2\frac{(1+\kappa^V_\rho)m^2_\rho}{6\pi^2Mg_Aq}\,e^{-q^2(1-a)/2\Lambda^2_\rho}[\bar{I}^0_1-\bar{I}^2_1]\,, \qquad \text{(C78)}
$$

$$
\begin{aligned}
\bar{I}^0_1 \;=\;& \int_0^{+\infty} dru^0_{00,1}(\kappa,r)j_0(bqr)[(H^2_{00}-H^2_{20})u_0(r) + (H^2_{22}-H^2_{02})u_2(r)/\sqrt{2}]\,, \qquad \text{(C79)} \\
\bar{I}^2_1 \;=\;& \int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,j_2(bqr)\,\Big[(H^2_{02}-H^2_{22})u_0(r) + \Big(-2H^2_{00} - H^2_{02} \\
&+H^2_{20}/5 + 16H^2_{22}/7 + 108H^2_{24}/35\Big)\,u_2(r)/\sqrt{2}\Big]\;, \qquad \text{(C80)} \\
b \;=\;& 1/2\,-\,a\,. \qquad \text{(C81)}
\end{aligned}
$$

These multipole contributions correspond to the part proportional to $\Delta_1\vec{j}^{\,a}_5(\rho\pi)$ of the current (A53). The contributions due to the part $\Delta_2\vec{j}^{\,a}_5(\rho\pi)$ can be obtained from Eqs. (C77) and (C78) by the change (A61).

3. The Δ excitation current of the π range $\vec{j}^{\,a}_{5\pi}(\Delta)$, Eq. (A62):

$$
<^1 S_0||i\hat{T}^{el}_{51}||d> \;=\; i\frac{\sqrt{2}g_AF_Am^3_\pi}{9\pi(M_\Delta-M)}\Big(\frac{f_{\pi N\Delta}}{m_\pi}\Big)^2\,(m^2_\pi-q^2_0)\Delta^\pi_F(q^2)
$$

$$
\begin{aligned}
& \times\left\{\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\pi}[j_2u_0(r)-(j_0+j_2/2)\sqrt{2}\, u_2(r)]\right. \\
& +\frac{1}{2}f(Z,Z)\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,(W_{2\pi}-3W_{1\pi}/x_\pi) \\
& \left.\times[-2j_0u_0(r)+\sqrt{2}j_2u_2(r)]\right\}\ , \qquad \text{(C82)}
\end{aligned}
$$

$$
\begin{aligned}
<^1S_0||\hat{L}_{51}||d> \;=\; & i\frac{2g_AF_Am_\pi^3}{9\pi(M_\Delta-M)}\left(\frac{f_{\pi N\Delta}}{m_\pi}\right)^2(m_\pi^2-q_0^2)\Delta_F^\pi(q^2) \\
& \times\left\{\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\pi}[-j_2u_0(r)+(j_2-j_0)u_2(r)/\sqrt{2}]\right. \\
& -\frac{1}{2}f(Z,Z)\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,(W_{2\pi}-3W_{1\pi}/x_\pi) \\
& \left.\times[j_0u_0(r)+\sqrt{2}j_2u_2(r)]\right\}\ . \qquad \text{(C83)}
\end{aligned}
$$

4. The Δ excitation current of the ρ range $\vec{\vec{j}}^{\,a}_{5\rho}(\Delta)$, Eq. (A64):

$$
\begin{aligned}
<^1S_0||i\hat{T}^{el}_{51}||d> \;=\; & -i\frac{\sqrt{2}g_AF_A\,m_\rho^3C_\rho^A}{9\pi(M_\Delta-M)}(m_\pi^2-q_0^2)\Delta_F^\pi(q^2) \\
& \times\left\{\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\rho}[j_2u_0(r)-(j_0+j_2/2)\sqrt{2}\, u_2(r)]\right. \\
& -4f(Y,Z)\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,(W_{2\rho}-3W_{1\rho}/x_\rho) \\
& \left.\times[j_0u_0(r)-j_2u_2(r)/\sqrt{2}]\right\}\ , \qquad \text{(C84)}
\end{aligned}
$$

$$
\begin{aligned}
<^1S_0||\hat{L}_{51}||d> \;=\; & -i\frac{2g_AF_A\,m_\rho^3C_\rho^A}{9\pi(M_\Delta-M)}(m_\pi^2-q_0^2)\Delta_F^\pi(q^2) \\
& \times\left\{\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\, W_{2\rho}[-j_2u_0(r)+(j_2-j_0)u_2(r)/\sqrt{2}]\right. \\
& -2f(Y,Z)\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,(W_{2\rho}-3W_{1\rho}/x_\rho) \\
& \left.\times\left[j_0u_0(r)+\sqrt{2}j_2u_2(r)\right]\right\}\ . \qquad \text{(C85)}
\end{aligned}
$$

5. The potential current of the ρ range $\vec{\vec{j}}^{\,a}_{5\rho}(p.c.)$, Eq. (A66):

$$
\begin{aligned}
<^1S_0||i\hat{T}^{el}_{51}||d> \;=\; & ig_AF_A\left(\frac{g_\rho}{2}\right)^2\left(\frac{m_\rho}{M}\right)^3\frac{(1+\kappa_\rho^V)^2}{8\sqrt{2}\pi} \\
& \times\left\langle\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\left[(qj_1+2j_2D^1_+)W_{1\rho}/m_\rho-j_2W_{2\rho}\right]u_0(r)\right. \\
& +\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\left\{[q(8j_1+3j_3)/5-2(2j_0+j_2)D^1_-]W_{1\rho}/m_\rho\right. \\
& \left.\left.+(2j_0+j_2)W_{2\rho}\right\}u_2(r)/\sqrt{2}\right\rangle\ . \qquad \text{(C86)}
\end{aligned}
$$

$$
\begin{aligned}
<^1S_0||\hat{L}_{51}||d> \;=\; & -ig_AF_A\left(\frac{g_\rho}{2}\right)^2\left(\frac{m_\rho}{M}\right)^3\frac{(1+\kappa_\rho^V)^2}{8\pi} \\
& \times\left\langle\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\left[(qj_1+2j_2D^1_+)W_{1\rho}/m_\rho-j_2W_{2\rho}\right]u_0(r)\right.
\end{aligned}
$$

$$+\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\Big\{[q(3j_3-7j_1)/5+2(j_0-j_2)D^1_-]W_{1\rho}/m_\rho$$
$$+(j_2-j_0)W_{2\rho}\Big\}\,u_2(r)/\sqrt{2}\Big\rangle\ . \tag{C87}$$

6. The time component of the weak axial MEC $\tilde{\rho}^a_5(\rho\pi)$, Eq. (A72):

$$\begin{aligned}
<^1 S_0||\hat{M}_{51}||d> \;=\; & i\Big(\frac{f_{\pi NN}}{m_\pi}\Big)^2\,\frac{m_\rho^2}{2\pi^2 g_A}\,e^{-q^2(1-a)/2\Lambda_\rho^2} \\
& \times\Big\langle a\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\Big\{[j_0(bqr)H^0_{00}+2j_2(bqr)H^0_{22}]u_0(r) \\
& +[j_2(bqr)H^0_{00}+(j_0(bqr)-j_2(bqr))H^0_{22}]\sqrt{2}\,u_2(r)\Big\} \\
& +\frac{1}{q}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,j_1(bqr)\Big\{(H^1_{01}+2H^1_{21})u_0(r) \\
& -[H^1_{01}+(H^1_{21}-9H^1_{23})/5]\sqrt{2}\,u_2(r)]\Big\}\Big\rangle\ .
\end{aligned}\tag{C88}$$

This multipole contribution corresponds to the part proportional to $\Delta_1\tilde{\rho}^a_5(\rho\pi)$ of the current (A72). The contribution due to the part $\Delta_2\tilde{\rho}^a_5(\rho\pi)$ can be obtained from Eq. (C88) by the change (A61).

Next follow the multipoles J=1 of the EFT MECs presented in Appendices B 1 and B 2.

3. Multipoles J=1 of the EFT MECs

1. The time component of the weak axial MECs, $\tilde{\hat{A}}^0_{2B}$, Eq. (B6),

$$<^1 S_0||\hat{M}_{51}||d> = -i\frac{g_A F_A m_\pi^2}{4\pi f_\pi^2}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\,j_1\,W_{1\pi}\,[u_0(r)-\sqrt{2}\,u_2(r)]\,. \tag{C89}$$

2. The $\tilde{\hat{\vec{A}}}_{2B}$ term, Eq. (B7),

$$\begin{aligned}
<^1 S_0||i\hat{T}^{el}_{51}||d> \;=\; & i\frac{g_A F_A m_\pi^3}{\sqrt{2}\pi M f_\pi^2}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\ \langle\{-(3/4+2\hat{c}_4+\hat{c}_3)j_0W_{1\pi}/x_\pi \\
& +[(3/4+2\hat{c}_4+\hat{c}_3)j_0+(\hat{c}_3-\hat{c}_4)j_2]W_{2\pi}/3+\Big[-2(j_0+j_2)D^1_+/3 \\
& +(1+c_6)qj_1]\,W_{1\pi}/4m_\pi-(\hat{d}_1+2\hat{d}_2)j_0W_\pi\Big\}\,u_0(r) \\
& +\{(3/4+2\hat{c}_4+\hat{c}_3)j_2W_{1\pi}/2x_\pi+[(\hat{c}_4-\hat{c}_3)j_0-(3/8+\hat{c}_4/2 \\
& +\hat{c}_3)\,j_2]\,W_{2\pi}/3+\Big[2j_0D^1_-/3+j_2(D^1_-+9D^3_+)/15 \\
& +(1+c_6)qj_1/2]\,W_{1\pi}/4m_\pi+(\hat{d}_1+2\hat{d}_2)j_2W_\pi/2\Big\} \\
& \times\sqrt{2}\,u_2(r)\Big\rangle\ ,
\end{aligned}\tag{C90}$$

$$<^1 S_0||\hat{L}_{51}||d> \;=\; i\frac{g_A F_A m_\pi^3}{2\pi M f_\pi^2}\int_0^{+\infty} dr\, u^0_{00,1}(\kappa,r)\ \langle\{-(3/4+2\hat{c}_4+\hat{c}_3)j_0W_{1\pi}/x_\pi$$

$$
\begin{aligned}
&+[(3/4+2\hat{c}_4+\hat{c}_3)j_0+2(\hat{c}_4-\hat{c}_3)j_2]W_{2\pi}/3+\Big[2(2j_2-j_0)D^1_+/3\\
&+qj_1\Big]\,W_{1\pi}/4m_\pi-(\hat{d}_1+2\hat{d}_2)j_0W_\pi\Big\}\,u_0(r)\\
&+\{-(3/4+2\hat{c}_4+\hat{c}_3)j_2W_{1\pi}/x_\pi+[(\hat{c}_4-\hat{c}_3)j_0\\
&+(3/4+2\hat{c}_3+\hat{c}_4)j_2]\,W_{2\pi}/3+\Big[2j_0D^1_-/3-2j_2(D^1_-+9D^3_+)/15\\
&-qj_1/2\Big]\,W_{1\pi}/4m_\pi-(\hat{d}_1+2\hat{d}_2)j_2W_\pi/2\Big\}\,\sqrt{2}\,u_2(r)\Big\rangle\;. \qquad \text{(C91)}
\end{aligned}
$$

3. The contribution due to the $\tilde{\hat{P}}(\vec{q}_1,\vec{q}_2)$ term, Eq. (B14),

$$
<^1S_0||\hat{L}_{51}||d>=i\frac{g_AF_Am_\pi^4}{\pi M f_\pi^2}\hat{c}_1\,\Delta_F^\pi(\vec{q}^{\,2})\int_0^{+\infty}dr\,u^0_{00,1}(\kappa,r)\,j_1\,W_{1\pi}\,[u_0(r)-\sqrt{2}\,u_2(r)]\,. \qquad \text{(C92)}
$$